# Controls and System Dynamics PhD Qualifying Exam Information Sheet and Instructions

## Objective

Modeling the dynamics of physical systems (both in terms of time and frequency domain models) and developing methods to control those systems. The system dynamics and controls qualifying exam tests fundamental concepts of system representation and control design based on that representation. In order to pass the exam, students must demonstrate a sound understanding of the topics listed below under "Exam Topics and Learning Objectives." This exam is meant to assess the PhD Program Outcomes of *subject mastery* and *independent learning*.

## Exam Instructions

1. Problem selection and grading:
    a. The exam will have 3 problems taken from the list of Exam Topics and Learning Objectives.
    b. You must complete 2 of the 3 problems.
    c. All 3 problems are weighted equally.
    d. Only the 2 problems to be graded should be handed in. If more than 2 problems are completed and handed in, the first of the 2 problems will be graded.
    e. A score of 70% or higher is considered a passing grade.
2. Procedures:
    a. The exam has a time limit of 2.0 hours.
    b. Each problem should be worked on a separate sheet of paper, with your name written at the top of each sheet.
    c. All work should be in neat engineering style with assumptions clearly stated.
3. Materials:
    a. The exam is open book and open notes.
    b. Solution manuals are not allowed.
    c. Calculators are allowed.
    d. Cell phones and other electronic devices are not permitted in the exam room.

## Exam Topics and Learning Objectives

***System Dynamics:***

| | |
|---|---|
| Representative course at BYU: | ME EN 335: Dynamic System Modeling and Analysis |
| Representative text: | William Palm III, *System Dynamics*, McGraw-Hill |

Learning Outcomes:

- **System Representation -** Obtain state variable equations from a set of nonlinear differential equations and know how to linearize nonlinear equations to obtain state space and transfer function representations.
- **Multi-domain Modeling** - Model and write differential equations for basic mechanical, electrical, fluid, and mixed systems (e.g. electro-mechanical, fluid/mechanical).

- **Poles/Eigenvalues** - Know how to manipulate transfer functions to find the poles of the system. Understand how to interpret poles and their effect on first-order (time constant) and second-order systems (natural frequency, damping ratio, time constant) including the expected time response (unforced and forced) and steady state behavior.
- **Frequency Response -** Understand the concept of frequency response for an LTI system and its relationship to transfer functions.

***Feedback Control:***

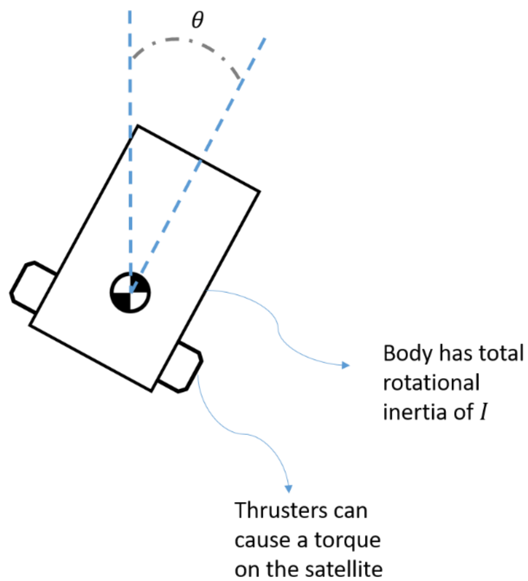
| | |
|---|---|
| Representative course at BYU: | ME EN 431: Design of Control Systems |
| Representative text: | Franklin et al., *Feedback Control of Dynamic Systems,* Pearson |

<u>Learning Outcomes:</u>

- **Stability -** Understand how the location of poles in the s-plane (or z-plane for discrete-time models) affect stability of a closed-loop system.
- **PID Control -** For linear time-invariant (LTI) models, use concepts of transfer functions, poles, rise time, settling time, and overshoot to guide the selection of gains for a PID controller to achieve satisfactory system response.
- **State Space Control Design -** Design gains for both full state feedback controllers and for state observers using state space models and pole placement techniques. Understand how introducing full-state feedback gains affects the poles and behavior of the system. Understand basic concepts of controllability and observability and how to test a given system.
- **Frequency Response -** Understand the concept of frequency response for an LTI system and its relationship to transfer functions.
- **Frequency Domain Control Design -** Design controllers using concepts from the frequency domain for LTI systems and understand their effect on phase and gain margin.

**<u>Sample Problem 1 – state feedback control and system stability:</u>**

The following matrix is the "A" matrix for a state space representation of the attitude motion of a satellite $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. The state for this system can be defined as $x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$ with a simple figure shown below.




Justify if this system is asymptotically stable or not. If the B matrix for the system is $B = \begin{bmatrix} 1/I \\ 0 \end{bmatrix}$, justify if the system is controllable or not.

If instead, $B = \begin{bmatrix} 0 \\ 1/I \end{bmatrix}$, justify if the system controllable.

For whichever B causes the system to be controllable, assume the rotational inertia $I = 1$, and find full state feedback gains that would give satisfactory performance of the system. Justify your response and be specific about what "satisfactory" means based on the performance of the system.

**Sample Problem 2 - manipulating transfer functions:**

a) Each of the transfer functions below represents a dynamic system. Find a value for the variable *a* which will make system A have the fastest free response.

A: $\frac{10}{s+2a}$

B: $\frac{7}{s^2+11s+10}$

C: $\frac{47}{s^2+3s+16}$

D: $\frac{37}{(s+7)(s^2+2.5s+1)}$

b) Find a value for the variable *a* which will make the system A have the slowest free response.

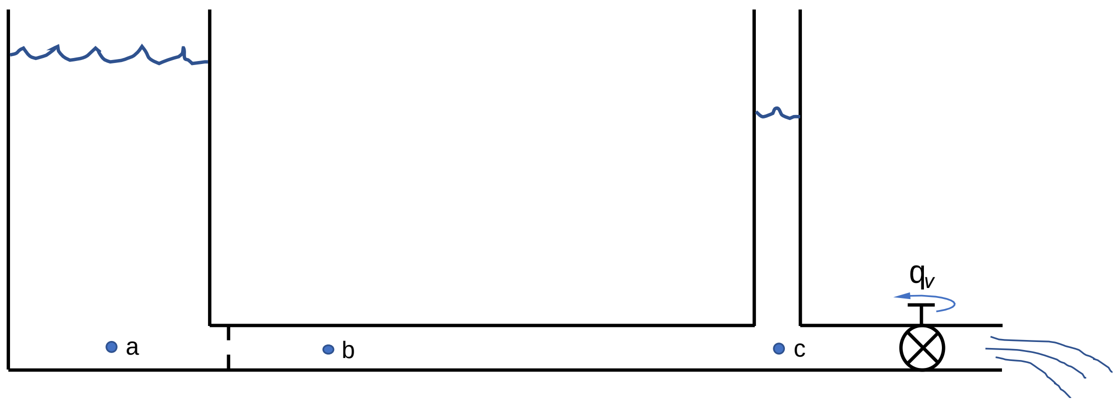
c) Sketch a plot of the time response for each of the above systems to a unit step input, assuming all initial conditions are 0.

d) An accelerometer has the transfer function shown below, relating acceleration to output voltage. You are looking for an accelerometer to measure the vibrations of a violin body in the audible range (20 Hz – 20 kHz). Do you recommend this accelerometer for the job? Why or why not? Include in your answer a sketch of the magnitude and phase plots for the accelerometer.

$$G(s) = \frac{9{,}375{,}000{,}000(s+10)}{(s+150{,}000)(s^2+2500s+625{,}000{,}000)}$$

**Sample Problem 3 - fluid system dynamic modeling:**

You are to model the dynamics of the fluid system shown below. Distinct pressures for this system are denoted by the points *a*, *b*, and *c* in the diagram. You are to model the fluid capacitance effects of the two tanks, the resistance of the orifice in the line, the fluid inertia in the line, and the resistance of the valve. For your model, you should assume that the fluid line is rigid and the fluid is not compressible. Derive the equations of motion. The resulting equations will have two pressure states and one flow state.



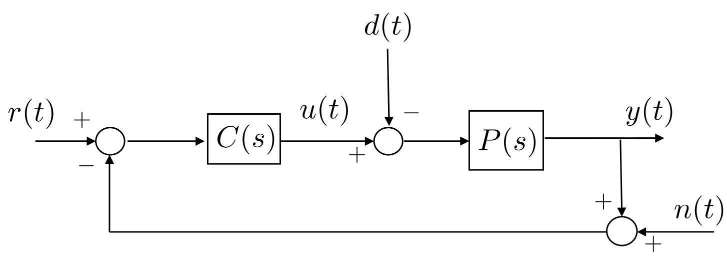

If you were interested in simulating the height of the fluid in the tanks, what would be your approach to do that?

**Sample Problem 4 - frequency-response-based control design:**

The rolling motion of an underwater vehicle is controlled by a PID controller of the form

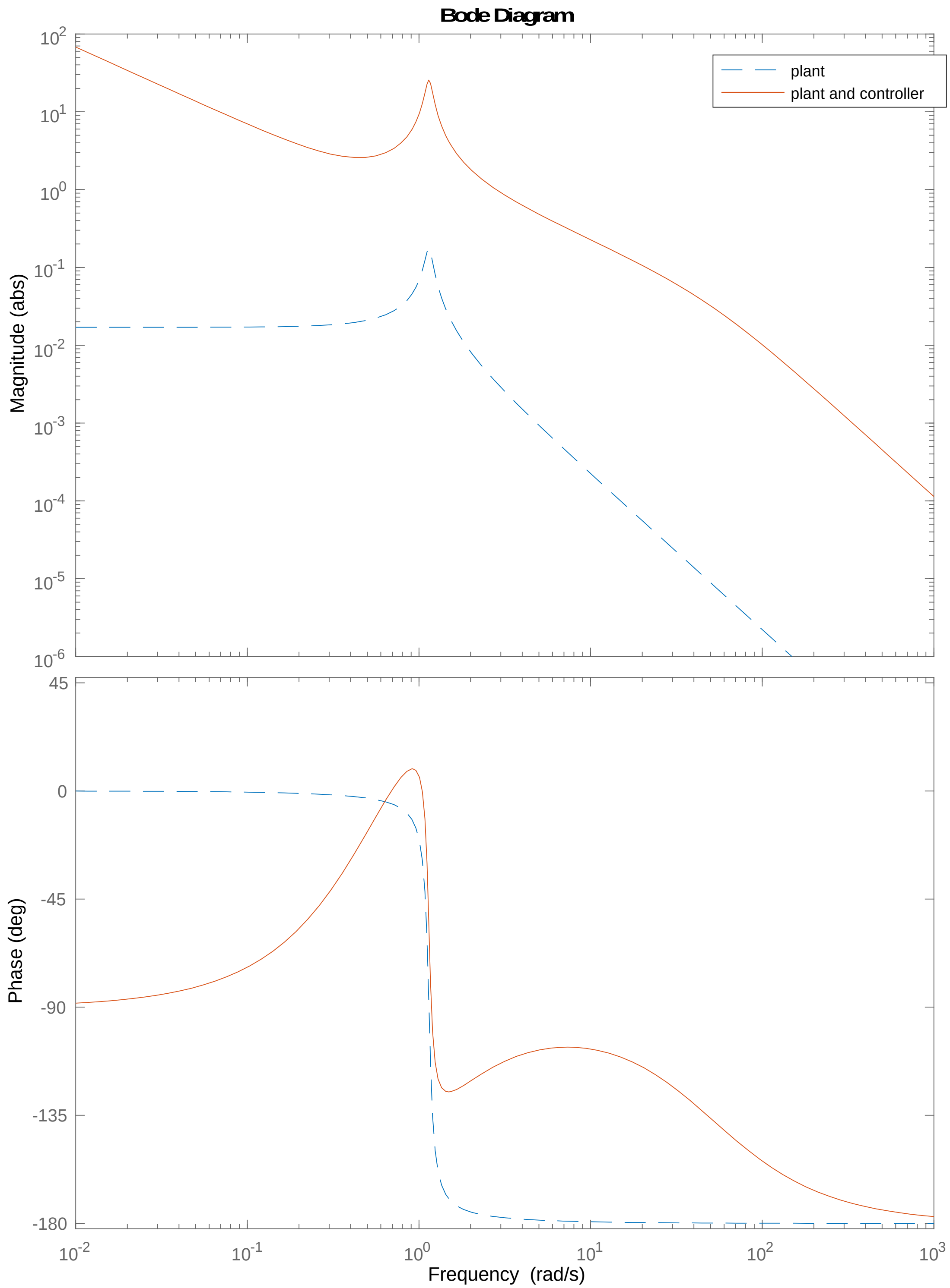
$$C(s) = k_p + \frac{k_i}{s} + \frac{k_d s}{\tau s + 1}$$

The control system has the following structure where $P(s)$ is the transfer function of the plant.




The *open-loop frequency response* for the plant $P(s)$ and the combined controller and plant $C(s)P(s)$ are shown on the following page. From these plots and the information above, determining the following:

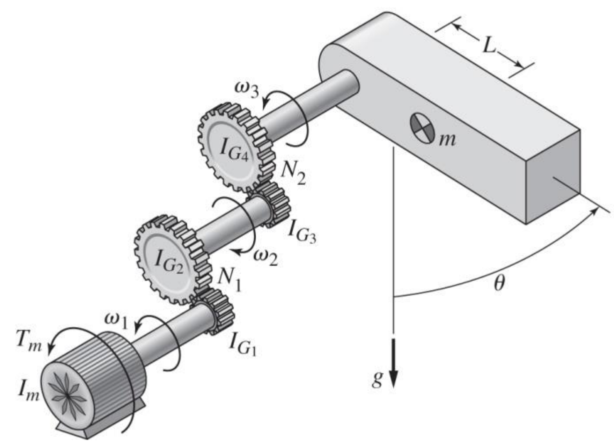
(1) Estimate the anticipated rise time for the closed-loop system in response to a step input in $r(t)$.
(2) Estimate of the anticipated percent overshoot for the closed-loop system in response to a step input in $r(t)$.
(3) Estimate the steady-state error of the system in response to a ramp input in $r(t)$.
(4) For a disturbance input given by $d(t) = 3\sin(0.2t)$, estimate the magnitude of the output $y(t)$.
(5) If noise is introduced by a sensor measurement that has a magnitude of 2 degrees, what is the lowest noise frequency that can be tolerated if the effect of the noise on the output must be kept below 0.01 degrees?


Bode Diagram
plant
plant and controller
Magnitude (abs)
Phase (deg)
Frequency (rad/s)
$10^{2}$
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
45
0
-45
-90
-135
-180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$

**Sample Problem 5: electro-mechanical modeling and system representation**

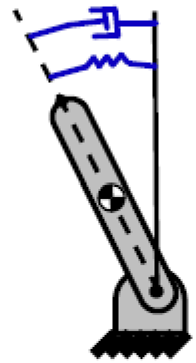
A permanent-magnet brushed DC motor drives a robot arm via a gear train, as shown in the figure. The input is the voltage $v_a$ applied to the armature of the motor, and the output is the arm angle $\theta$. $N_1$ and $N_2$ represent the gear ratios. The inertias of all elements are indicated on the figure. Do the following, assuming no mechanical damping or friction for the load:

1. Select an appropriate set of state variables and derive a state variable model of system. In your motor model, include the motor's inductance, inertia, back-emf, damping, electrical resistance, and torque constant. Provide a justification of why you selected a particular number of state variables, and why you selected particular states.
2. Assuming $\theta$ remains small, find a transfer function relating the output, $\Theta(s)$, to the input, $V_a(s)$.
3. If the shaft between $I_{G\,4}$ and the arm were flexible, how many states would be required for the system? Why? What would you choose as the states? Why?



Given the differential equations below that describe an inverted pendulum with a nonlinear spring, viscous friction, and a motor that applies torque,

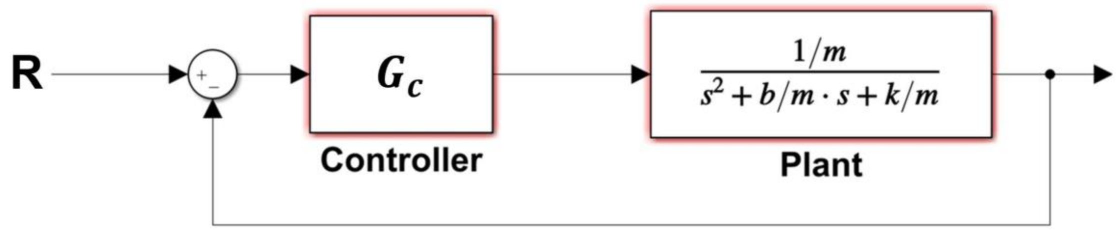
$$I\ddot{\theta} + b\dot{\theta} + mgLsin(\theta) + k(1 + a^2\theta^2)\theta = \tau$$



please do the following:

a) Find a linearized state-space representation of this system with states $\theta$ and $\dot{\theta}$ for the equilibrium point when these both equal zero and no torque is being applied.
b) Perform a stability analysis on the linearized system and also show if the system is controllable.
c) Explain conceptually what the stability of the linearized system tells us about the original nonlinear system.
d) Using full-state feedback for the linearized system, assuming it is an underdamped system, find a set of gains (K), that will improve the rise time of the original system to be twice as fast.

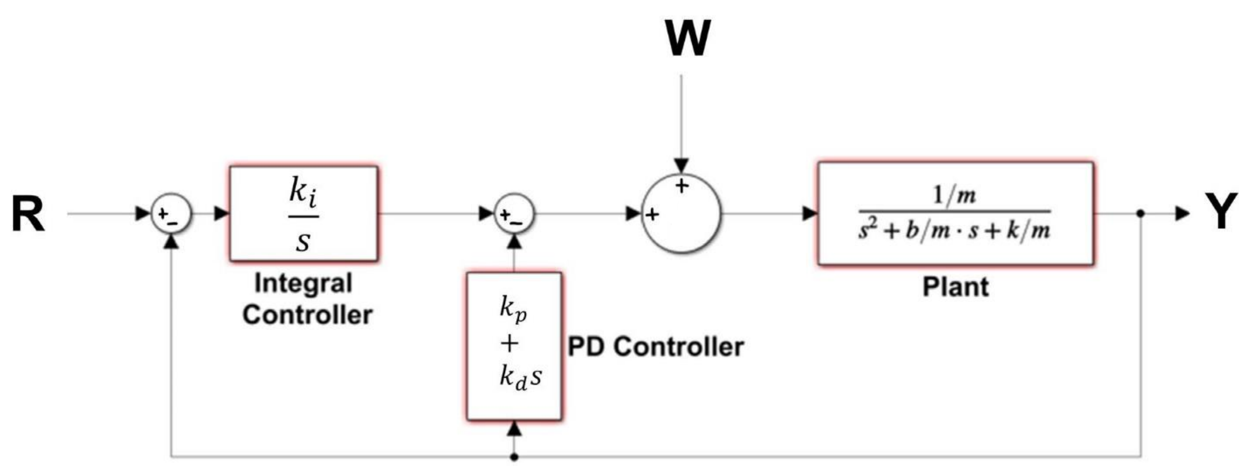
Given the following block diagram and transfer function for the plant:



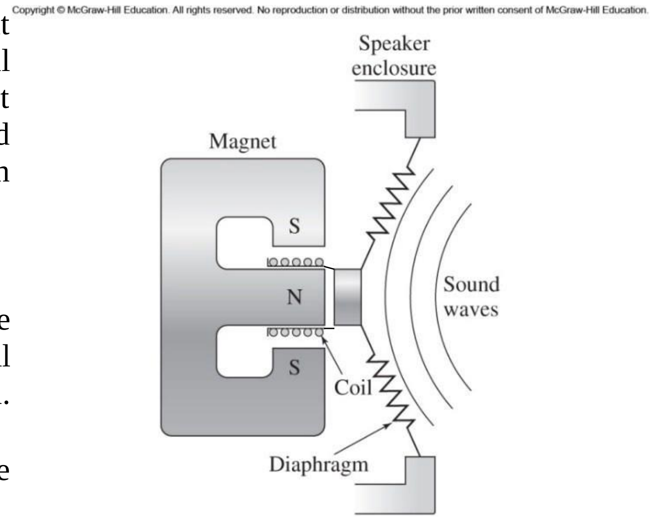

a) If we let the controller $G_c$ be a simple PD controller, what is our transfer function between the reference input (R) and the plant output (Y)?

b) Neglecting the effect of any zeros in the transfer function from part a), pick gains ($k_p$ and $k_d$) that will give a rise time of 0.09 sec and a settling time of 0.46 sec if m = 1kg, b = 10 Ns/m, and k = 100 N/m.

c) If instead we chose the following block diagram for our controller architecture (where W is a disturbance input), what will our steady state value of $y$ be for a unit-step input in the reference $r$? What about for a unit step in w?



Many microphones consist of a permanent magnet, a coil, and a diaphragm, with the coil attached to the diaphragm. As sound waves hit the diaphragm, they move the diaphragm and coil relative to the permanent magnet, which generates current in the coil.




a) If the input to this system is the force, $f_s$, exerted by the sound waves onto the diaphragm, and if the output is the coil current, $i$, develop a model of the system. Note the following:
   - The force, $f$, exerted by the permanent magnet onto the coil can be approximated as a linear function of $i$: $f = K_f i$, where $K_f$ is the torque constant.
   - The back-emf, $v_b$, generated in the coil is proportional to the velocity, $\dot{x}$, of the coil and diaphragm: $v_b = K_b \dot{x}$, where $K_b$ is the back-emf constant
   - You may assume that the movements of the diaphragm and attached coil are relatively small.

b) Derive the transfer function of this system.

c) Develop an expression for the frequency response (magnitude ratio and phase shift) of the system.

d) A particular microphone, which has the parameter values in the table below, is used to record a duet by a male and a female singer. Will the recorded signal accurately represent the duet? Why or why not? For simplicity, assume that the male and female singers' voices each consist of a single tone at 100 Hz and 200 Hz, respectively.

| | |
|---|---|
| Mass of diaphragm and coil | 0.002 kg |
| Diaphragm damping | Negligible |
| Diaphragm stiffness | 400,000 N/m |
| Torque constant | 16 N/A |
| Back-emf constant | 13 Vs/m |
| Coil resistance | 12 Ω |
| Coil inductance | 0.001 H |