Heat Transfer PhD Qualifying Exam
Information Sheet and Instructions

Objective:
Students should demonstrate a satisfactory understanding of heat transfer exam topics as outlined below. Additionally, a satisfactory understanding includes the ability to apply and utilize fundamental principles and concepts to solve heat transfer problems using appropriate assumptions and connections. Most undergraduate texts contain problems ranging from simple questions intended to reinforce the understanding of basic principles to more advanced problems requiring a synthesis of the principles and concepts and their applications to various engineering applications. Students should prepare for the qualifier by mastering these more advanced problems.

Exam Format:
- This exam is comprised of problems that can span the primary heat transfer topics of conduction, convection and radiation (further details on topics are given below).
- This exam is open book and open notes with the exception of solution manuals of any kind, which are not allowed. Calculators may be used. Internet-connected or other communication devices are not permitted in the exam room. However, if access to an e-text is required, all communication methods must be disabled (e.g., wifi, Bluetooth, etc.)
- All work should be in neat engineering style with assumptions clearly stated. Partial credit may be awarded, but only in cases where the assumptions and solution approach are clearly explained in response to the questions. Start each problem solution on a separate sheet of paper.
- The exam has a time limit of 2 hours.
- A score of 70% or higher is considered a passing grade.

Exam Topics:
For general exam preparation, review the ME Department *PhD Qualifying Exam General Information* document. The topics for the heat transfer qualifying exam focus on the three primary areas of heat transfer: (1) conduction, (2) convection, and (3) radiation. Study of these heat transfer topics in preparation for the exam may include completing and reviewing material from an undergraduate heat transfer course (e.g., ME EN 340 at BYU or equivalent) and reviewing heat transfer principles as found in the following reference or similar undergraduate texts: Bergman, T. L., Lavine, A. S., Incropera, F. P., and DeWitt, D. P., 2011, *Fundamentals of Heat and Mass Transfer*, 7 ed, John Wiley and Sons, Inc.

The following outline of topics provides additional clarity on the scope of exam topics. Concepts provided here are NOT intended to be exhaustive but represent broad topics in the discipline of heat transfer that the qualifying exam will include.

1. *Conservation Principles* – Each student can build models of multimode heat transfer processes and systems by applying conservation of mass and energy to a system.

2. *Fundamentals of Conduction* - Each student can describe the physical mechanisms involved in conduction heat transfer. Each student can use Fourier's law in conjunction with conservation of energy to develop the heat diffusion equation.

3. *Conduction Analysis* - Each student can utilize solution methods for the heat diffusion equation to analyze 1D, 2D, steady and transient problems, including the use of thermal circuits and analytical and numerical methods.

4. *Extended Surfaces* - Each student can analyze extended surfaces using the fundamentals of conduction and convection. Each student can use fin efficiency and fin effectiveness to evaluate the performance of a fin or fin array.

5. *Fundamentals of Convection* - Each student can describe the physical phenomena associated with convection and use non-dimensional parameters to analyze convection heat transfer. Each student can calculate local and global convective heat fluxes using Newton's law of cooling.

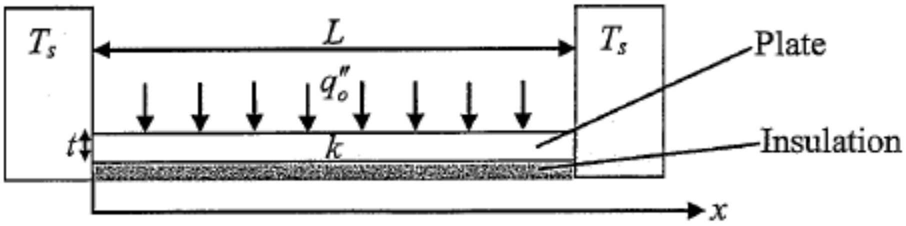
6. *Convection Analysis* - Each student can use empirical correlations to analyze external and internal, forced and free convection problems.

7. *Fundamentals of Radiation* - Each student can describe the physical mechanisms involved in radiation heat transfer. Each student can model radiative heat transfer processes and include radiative processes when analyzing heat transfer at a surface.

8. *Radiative Heat Exchange* - Each student can calculate total, hemispherical radiative properties of surfaces from their spectral, directional counterparts and evaluate the radiative heat exchange between diffuse, gray surfaces in enclosures.

The following pages provide sample problems that are representative of the type anticipated for the qualifying exam.

A thin flat plate of length *L*, thickness *t*, and width (into the page) *W* is positioned between two heat sinks in an evacuated chamber as illustrated below. The thermal conductivity of the plate is *k*. Note that *W>>L>>t*.



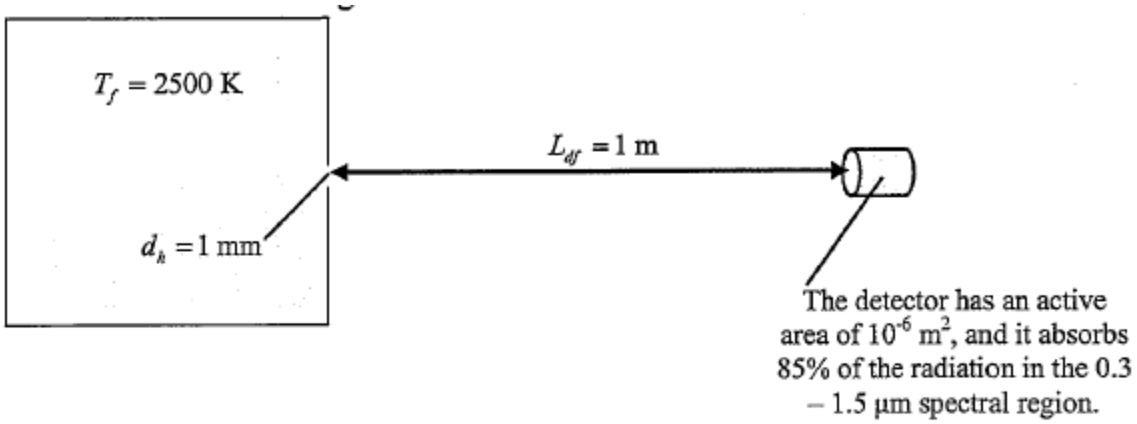
The heat sinks are maintained at a temperature of T_S, and the bottom of the plate is well insulated. The upper surface of the plate is exposed to a known, uniform heat flux, q''_0. Develop a model that may be solved for the steady state temperature profile in the plate, *T(x)*. Clearly state any approximations used to develop the model. Solve the model for the temperature profile and calculate the rate heat is transferred to the heat sinks.

Consider transient heat conduction to a sphere with its outer surface held at an elevated constant temperature. Derive an expression for the ratio of the total energy transferred into the sphere (*Q*) over some time interval *t* to the maximum possible energy transfer (Q_o) that is valid for *Fo* < 0.2. Express your result in terms of the Fourier number, *Fo*. Identify the limiting value for Q/Q_o as Fourier number approaches 0.2.

Develop an expression for the ratio of the total energy transferred into the sphere (*Q*) over some time interval *t* to the maximum possible energy transfer (Q_o) that is valid for *Fo* ≥ 0.2. Express your result in terms of the Fourier number, *Fo*. Identify the limiting value for Q/Q_o as Fourier number approaches 0.2.

Would you expect the limiting values of Q/Q_o as Fourier number approaches 0.2 to be similar for the two cases? Why or why not?

A detector is placed 1 m from a 1 mm diameter hole in the side of a 2500 K furnace as illustrated in the figure below.

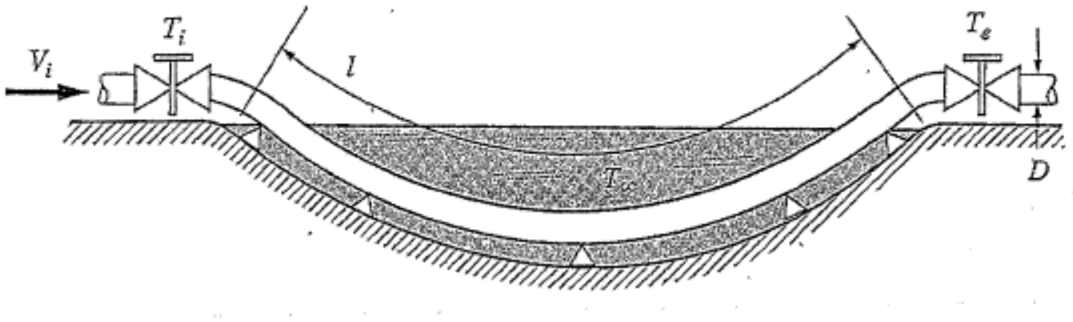


The output from the detector is an electrical current which is proportional to the absorbed radiation,

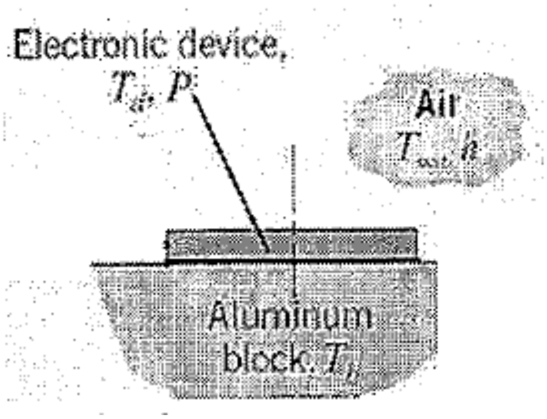
$$S = Cq_d$$

Where *S* is output signal in mA, *C* is the constant of proportionality and q_d is the rate energy is absorbed by the detector in mW. Neglecting radiation from the surroundings, determine the signal output by the detector if *C*=15 mA/mW.

Ethylene glycol is pumped through a pipeline of diameter *D*=0.4 m which runs across a lake *l*=200 m wide. The velocity and inlet temperature of the oil are 2.5 m/s and T_i=50°C. The temperature of the lake water is $T_\infty = 5^oC$. Evaluate the exit temperature of the oil. State and justify all assumptions.

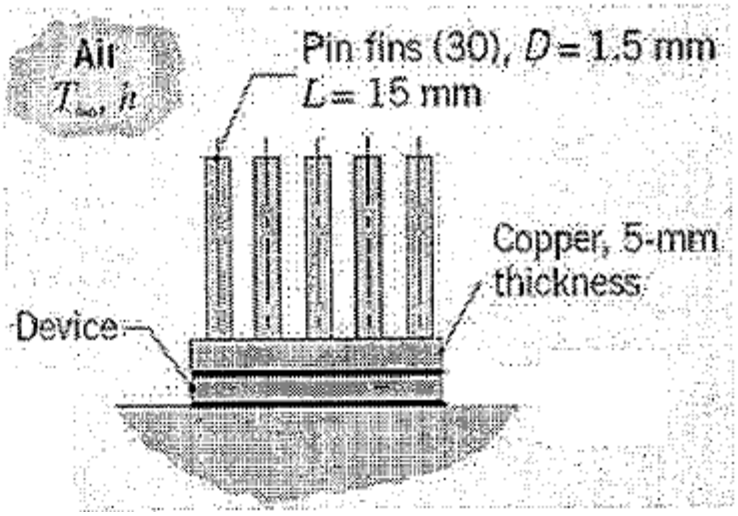


An electronic device, which is a disk with a diameter of *d*=25 mm, consumes power at a rate of *P*. As shown in the figure, the device is flush mounted on an aluminum heat sink (*k*=177 W/m-K) that is maintained at a constant temperature of 27 C. The device is also exposed to a convective environment in which $T_\infty = 27\ C$ and $h = 10^3\frac{W}{m^2} - K$.



A. What is the rate at which the device consumes power, *P*, if the temperature of the device is $T_d = 75\ C$?

To allow the device to be operated at a higher power level, a circuit designer suggests attaching a finned heat sink to the top of the device as shown below. The pin fins and the base are made from copper (*k*=400 W/m-K) and are exposed to the same convective environment ($T_\infty = 27\ C$ and $h = 10^3\frac{W}{m^2} - K$)



B. If the finned heat sink is included in the design and the device operates at $T_d = 75\ C$, what is the allowable increase in the power consumption? Would you recommend including the finned heat sink in the design?

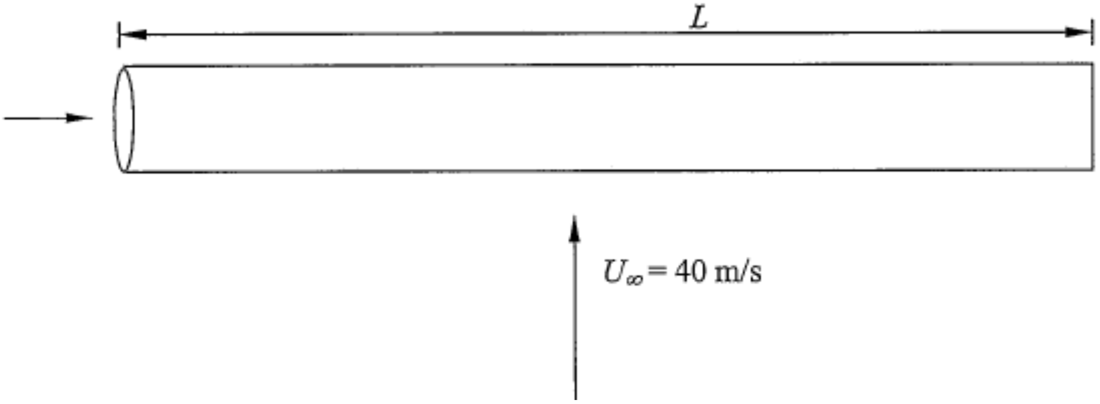
Stream at atmospheric pressure is passing through a long heat exchanger tube in an industrial furnace. At the inlet of the tube the stream has a mixed mean temperature of 500 K. The inside diameter of the pipe is 0.1 m and the stream is moving at a speed of 60 m/s. On the outside of the tube products of combustion (assumed to have the same thermophyiscal properties as air at atmospheric pressure) are passing over the tube (normal flow direction as shown below) at a speed of 40 m/s with a temperature that varies approximately linearly from 1100 K at the mid-section of the tube (*x=L/2*) to 750 K at the upstream and downstream ends of the tube (*x*=0 and *x*=*L*). The outer diameter of the pipe is 0.11 m and the pipe is fabricated from stainless steel (AISI 302). State and justify all assumptions in your analysis.

A. (70%) What is the mixed-mean steam temperature at a distance of *L=5* m downstream from the inlet?

B. (20%) At what angular location (measured from the stagnation point) is the external surface temperature on the pipe a minimum and where is it a maximum? Estimate these temperature values at *x*=0, 2.5 m, and 5 m.

   Note: In the above analysis conjugate conduction (heat transport along the length of the pipe through the pipe wall due to conduction) may be neglected.

C. (10%) Now conduct an engineering analysis regarding the relative magnitude (with respect to the thermal transport in the wall-normal direction) of the conjugate heat transfer in the pipe material and determine whether or not neglecting it is appropriate in this analysis.



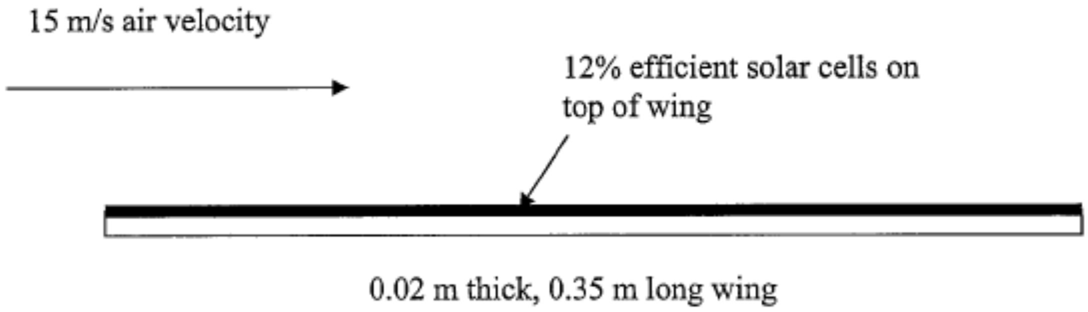
Undergraduate Mechanical Engineering students at BYU are building a solar powered airplane. The wing of the plane is covered with solar cells. The cells we are using are 12% efficient at converting the sun's energy into useful electricity. The remaining 88% of the sun's energy can be modeled as energy either reflected from the cells, or absorbed by the cells. The energy absorbed by the solar cells causes them to become hot. The hotter the cells become, the lower their efficiency. We want to maintain the solar cells as cool as possible.

Assume the wing is a flat plant that is 0.35 m long (wing chord) and 4.75 m wide (wing span). Assume the plane is flying at 15 m/s. The solar cells are mounted on the top of the wing. The wing can be modeled as a 0.02 m thick insulating panel. The solar constant has a value of about 1353 W/m^2 outside the earth's atmosphere. On a typical day in Provo, this is reduced to about 800 W/m^2 by the earth's atmosphere.

You will need to make some assumptions about environment and material properties to solve this problem. List your assumptions.

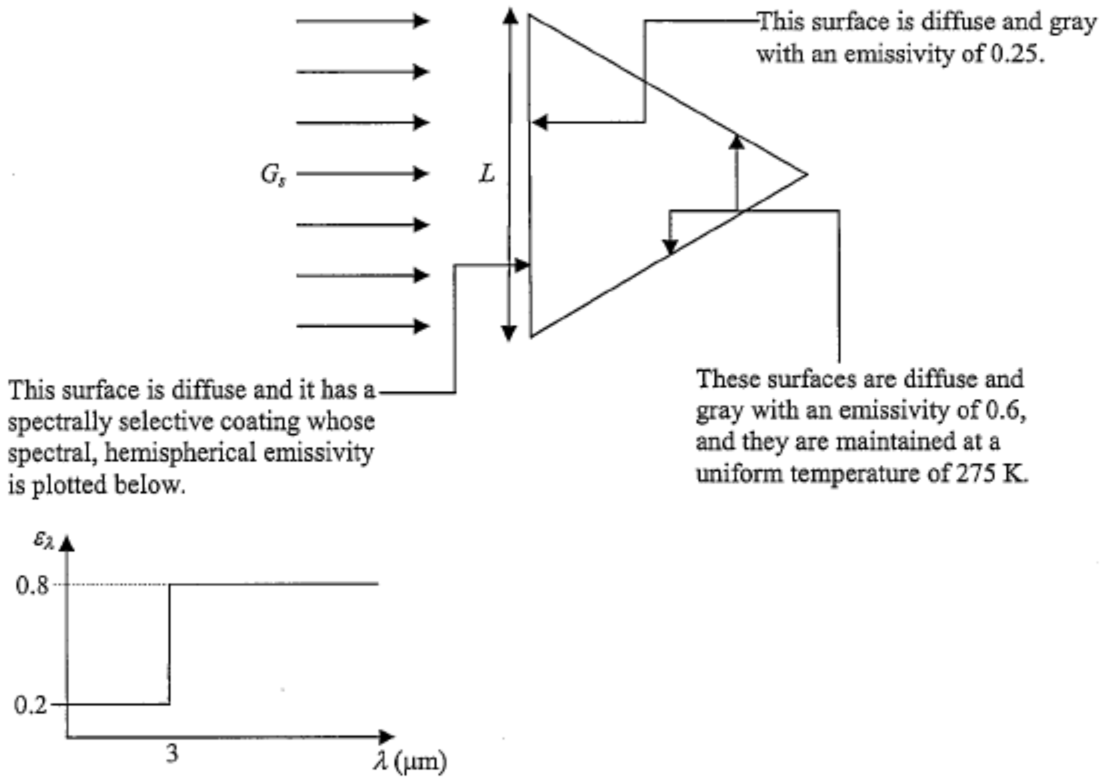
A. Estimate the equilibrium temperature reached by the solar cells at the worst time of day and under the worst conditions.

B. Propose methods to lowering this temperature.



The outer surface of a long 5.0-cm-diameter copper (386 W/m°C) cylinder is maintained at 200°C when exposed to room air at 20°C with h=3.0 W/m^{2}K. The cylinder is wrapped in asbestos insulation (k=0.17 W/m°C) to reduce heat transfer from the cylinder. What insulation thickness is required to reduce the heat transfer to ½ of its original value?

A spacecraft is flying 1.75x10^{11} m from the sun. A component of the spacecraft consists of an enclosure formed by three long plates of equal width (L=1 m) as illustrated below. The outer surface of the plate that is exposed to direct solar radiation is diffuse and it has been given a spectrally selective coating. The inner surface of the exposed plate is diffuse and gray, as are the other plates. These other plates are maintained at a uniform temperature of 275 K. The diameter of the sun is 1.39x10^9 m, and the sun emits like a blackbody at 5800 K.



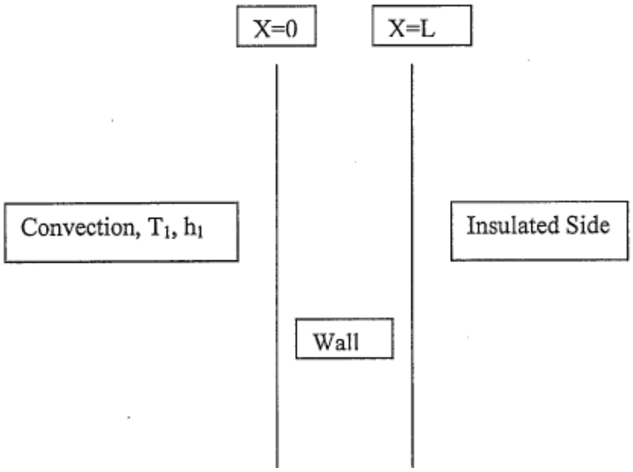
A. What is the temperature of the plate that is exposed to solar irradiation?

B. What is the rate, per unit length into the page, at which heat flows through the other plates?

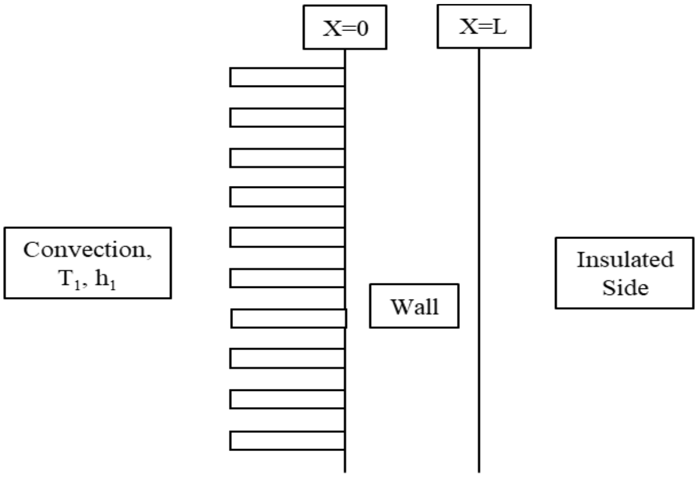
The wall of a nuclear reactor is cooled on the inside by convection to water. The temperature of the water is T_1 and the convection heat transfer coefficient from the wall to the water is h_1. The outside of the reactor wall is insulated. The wall has thickness L and thermal conductivity k. Neutrons leaving the reactor core are absorbed in the reactor wall. This results in energy generation in the wall. The energy generation is not uniform across the wall, but decreases exponentially from the inside of the wall to the outside. The energy generation per unit volume can be describe by the equation:

$$\dot{q}(x) = \dot{q}_o e^{(-ax)} (W/m^3)$$

Derive an expression for the maximum temperature in the wall. Sketch the temperature distribution in the wall.



The reactor in the last problem is modified by adding fins to the convective side of the wall. The fins increase the surface area of the wall by a factor of 10. Assume the convective heat transfer coefficient is not affected by the fins. How will the maximum temperature in the wall be affected by the fins? Sketch the old and new temperature distributions in the wall. As much as possible, quantify the difference between the old and new temperature distributions.



A flat surface on an earth orbiting satellite is coated with spectrally selective black nickel. Black nickel is a diffuse emitter, and its spectral emittance may be approximated by

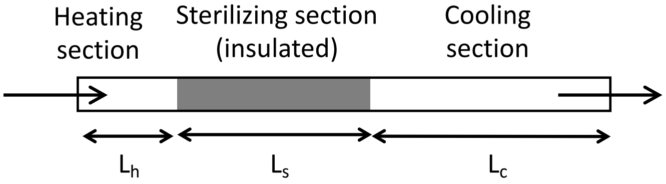
$$\varepsilon_\lambda = \begin{cases} 0.9, & \lambda < 2\mu\text{m} \\ 0.25, & \lambda > 2\mu\text{m} \end{cases}$$

Radiation coming from the earth and from stars other than the sun is negligible. The following data is available.

- The sun radiates like a blackbody at 5777 K.
- The radius of the sun is 6.96×10^8 m.
- The distance from the sun to the earth is 1.496×10^{11} m.

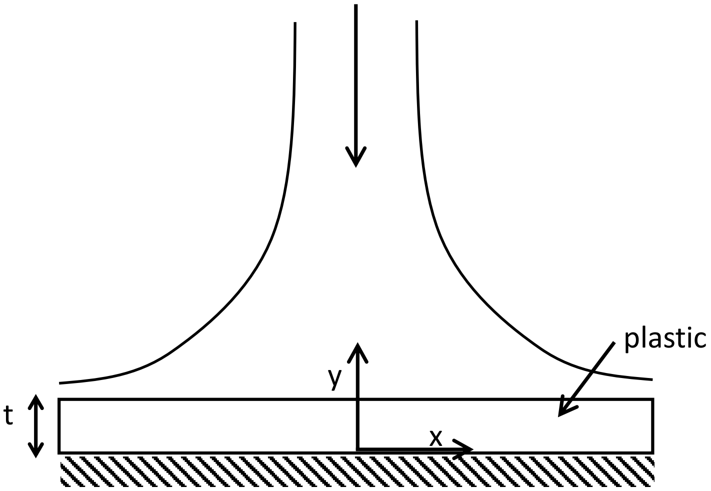
Calculate the equilibrium temperature of this surface as a function of the angle between its normal and the sun's rays. What is the equilibrium temperature of the surface if it is oriented normal to the sun's rays? What is the equilibrium temperature if the surface is oriented 60° from the sun's rays?

A liquid food product is processed in a continuous flow sterilizer. The liquid enters the sterilizer at a temperature and flow rate of $T_{m,i,h}$ = 20 °C, $\dot{m} = 1$ kg/s, respectively. A time-at-temperature constraint requires that the product be held at a mean temperature of T_m = 90 °C for 10 s to kill bacteria, while a second constraint is that the local product temperature cannot exceed T_{max} = 230 °C in order to preserve a pleasing taste. The sterilizer consists of an upstream, L_h = 5 m heating section characterized by a uniform heat flux, an intermediate insulated sterilizing section, and a downstream cooling section of length L_c = 10 m. The cooling section is composed of an insulated tube exposed to a quiescent environment at T_∞ = 20 °C. The thin-walled tubing is of diameter D = 40 mm. Food properties are similar to those of liquid water at T = 330K.



A. What is the heat flux required in the heating section to ensure a maximum mean product temperature of T_m = 90 °C?
B. Determine the location and value of the maximum local product temperature. Is the second constraint satisfied?
C. Determine the minimum length of the sterilizing section needed to satisfy the time-at-temperature constraint.
D. Sketch the axial distribution of the mean, surface, and centerline temperatures from the inlet of the heating section to the outlet of the cooling section.

A molded plastic product (ρ = 1200 kg/m^3, c = 1500 J/kg·K, k = 0.3 W/m·K) is cooled by exposing the upper surface to a slotted air jet (resulting in a 2D scenario), while the opposite surface is well insulated. The product may be approximated as a plate of thickness t = 60 mm. Initially the transistor is at a uniform temperature of T_i = 80 °C. The impinging air jet (T_∞ = 20 °C) is then turned on to cool the transistor and provides a spatially varying convection coefficient of $h = 100\, e^{(-x/2)}$ W/m^2·K (for x>0) at the cooled surface.



Develop a numerical approach to determine the temperature through the thickness of the plastic at the stagnation point after 1 h of exposure to the gas jet.

Write the discretized form of the energy balance for each unique set of conditions required to solve for this scenario. Do not solve.
