Dynamics PhD Qualifying Exam
Information Sheet and Instructions

Objective

Mechanical engineering has a focus on the motion of systems and the forces or moments that cause that motion. The dynamics qualifying exam tests fundamental concepts of motion and force for mechanical engineers working in the areas of classical dynamics or mechanics. Students must demonstrate a firm grasp of the fundamental principles of classical dynamics and lumped parameter modeling as defined in the Topics and Learning Objectives section of this document in order to pass the exam.

Instructions

1. Problem selection and grading:
    a. The exam will have 3 problems taken from the list of Topics and Learning Objectives.
    b. You must complete 2 of the 3 problems.
    c. All 3 problems are weighted equally.
    d. Only the 2 problems to be graded should be handed in. If more than 2 problems are completed and handed in, the first of the 2 problems will be graded.
    e. A score of 70% or higher is considered a passing grade.
2. Procedures:
    a. The exam has a time limit of 2.0 hours.
    b. Each problem should be worked on a separate sheet of paper, with your name written at the top of each sheet.
    c. All work should be in neat engineering style with assumptions clearly stated.
3. Materials:
    a. The exam is open book and open notes.
    b. Solution manuals are not allowed.
    c. Calculators are allowed.
    d. Cell phones and other electronic devices are not permitted in the exam room.

Exam Topics and Learning Objectives

Problems will be selected from the general area of classical dynamics. Representative courses, texts, and topics for each of these areas are given below.

Representative courses at BYU: CE EN 204: Engineering Mechanics – Dynamics

ME EN 335: Dynamic System Modeling and Analysis

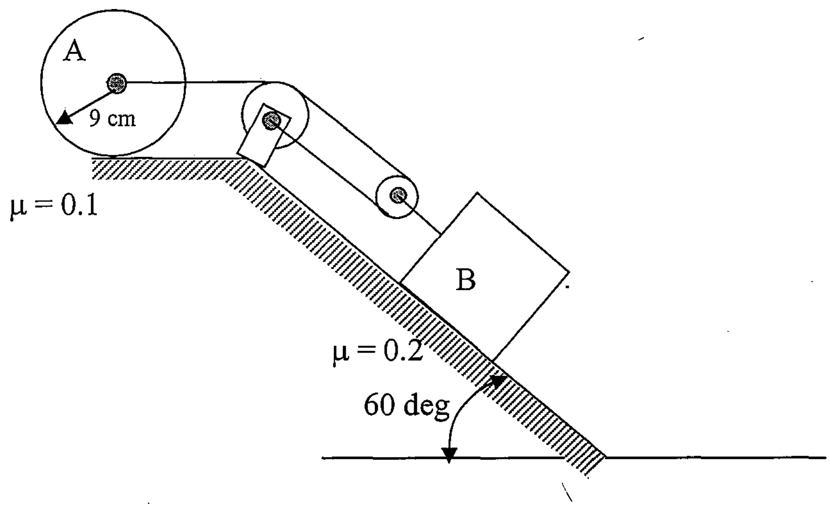
Representative texts: R.C. Hibbeler, *Engineering Mechanics: Dynamics*, Pearson

William Palm III, *System Dynamics*, McGraw-Hill

Learning Outcomes:

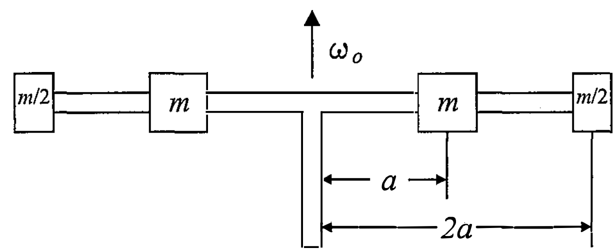
- **Kinematics** – Understand the kinematics of particles and planar rigid bodies, and express motion in appropriate coordinate systems.
- **Laws of Motion** – Use Newton's Laws of Motion to write and solve equations of motion for particles and systems of particles.
- **Energy and Momentum** – Understand the principles of Work and Energy and Impulse and Momentum and solve problems using these principles. Be able to apply these principles to solve impact problems.
- **Motion of Rigid Bodies** – Model and solve for the motion of rigid bodies in general planar motion.
- **Mechanical Systems** – Know fundamental systems concepts required to develop lumped element models for basic mechanical systems, including inertia, compliance, dissipation, and input forces or torques, and obtain equations of motion.
- **Vibration** – Understand the basic concepts of one-degree-of-freedom vibration and solve simple problems related to such motion.

Cylinder A has a mass of 7 kg and is attached to the 8 kg block B using the cord and pulley system shown. Determine the initial angular acceleration of cylinder A after block B is released. The coefficients of static friction and other parameters are indicated in the figure.

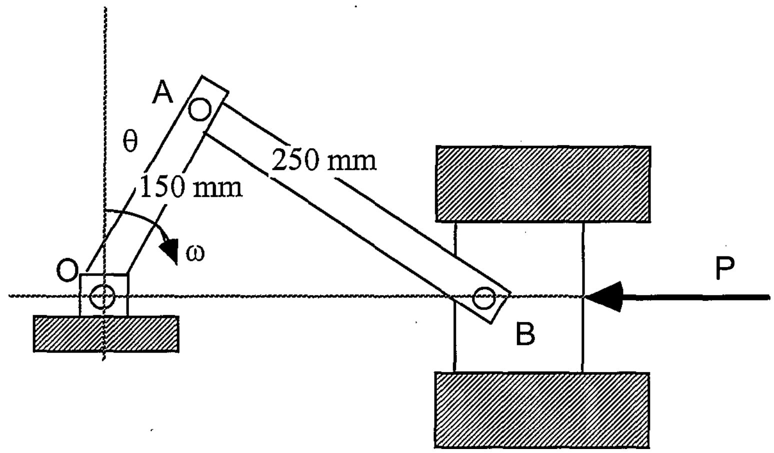


A frictionless rod is rotating freely with an initial angular speed ω_0 about a vertical axis. The two sliders, each of mass *m*, are released at a radial distance $r = a$. The rod is massless. The stops at the ends of the rod each have a mass $m/2$.

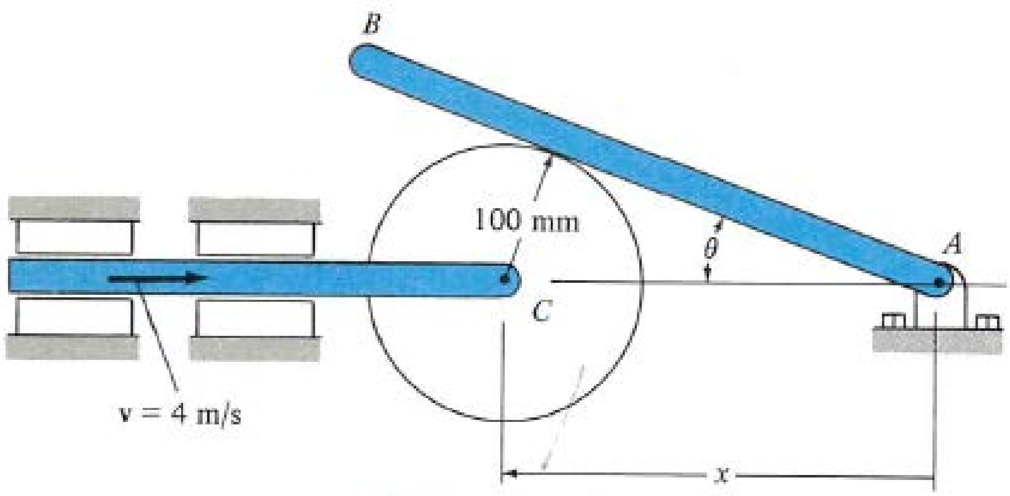
a. Find the angular speed of the system after the sliders come to rest at the stops.
b. Find *and explain* the change in kinetic energy of the system.



The 5 kg crank arm (OA) is rotating at a constant speed of $\omega = 100$ rpm. The uniform piston rod AB has a mass of 6.3 kg and at $\theta = 67°$ the piston is subject to a piston chamber force of P = 800 N. If the piston has a mass of 5.5 kg, what are the reaction forces at joint A? Neglect any friction forces. Please show your free-body diagrams and your work in detail.

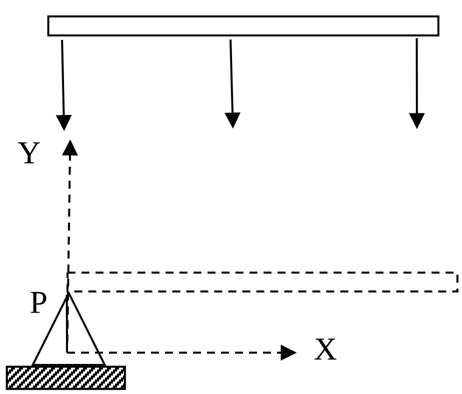


Compute the angular velocity and angular acceleration of uniform rod AB when $\theta = 30^{o}$, assuming that the horizontal shaft moves the CAM cylinder C at a constant speed of 4 m/s from left to right. Now if you were asked to determine whether rod AB reaches a "relative" maximum angular acceleration between the angles of $\theta = 20^{o}$ and $\theta = 40^{o}$, how would you do that ("relative" here means a local maximum where the time derivative of acceleration is zero, but may not be the largest possible maximum)?

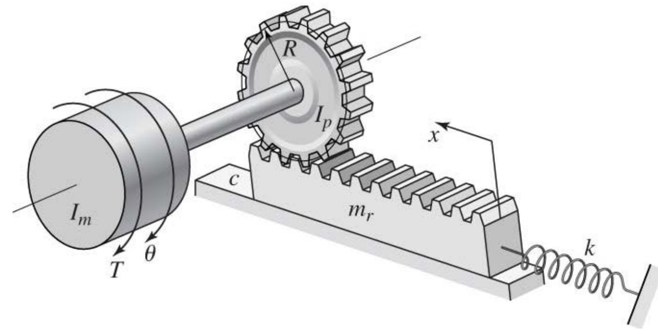


A uniform bar of length L = 1 m is moving in a zero gravity, zero atmosphere field at a uniform speed of 10 m/s as shown by the arrows. Initially parallel to the X direction, the bar is not rotating when it strikes a fixed object at the left end shown as point P. When it strikes the object at P, assume that the collision is fully plastic at the contact point P (no bar rebound or relative sliding) so that motion of the bar immediately after impact is a pure rotation about the object (model P as a pinned joint).

1. What is the angular velocity of the bar immediately (small time period) after impact?
2. In what direction is the impulsive force applied to the fixed object during the early impact?
3. What are the kinetic energies of the bar both before and after impact if the mass of the bar is 1 kg?
4. Are the kinetic energies before and after impact different? If so, explain the difference?



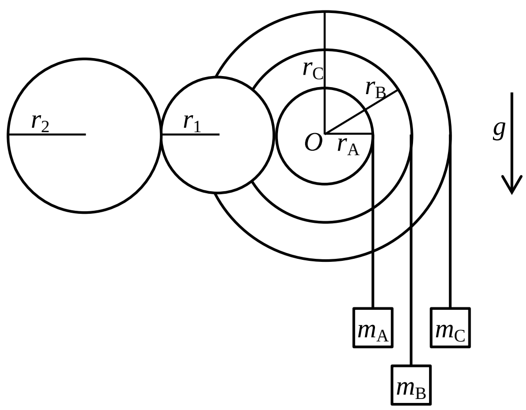
As shown in the figure, a torque T is applied to a rotor that has inertia I_m. A pinion of radius R and inertia I_p drives the rack of mass m_r. A damping force (with damping coefficient c) and a spring (with spring constant k) act on the sliding rack. The shaft connecting the rotor to the pinion is flexible with an effective rotational spring constant k_s. The shaft's mass is very low compared to the rotor and pinion gear.

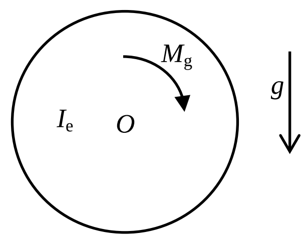


Find the equations of motion of the system in terms of x (the displacement of the rack) and θ (the angle of rotation of the rotor). The displacements are measured from the static equilibrium positions.

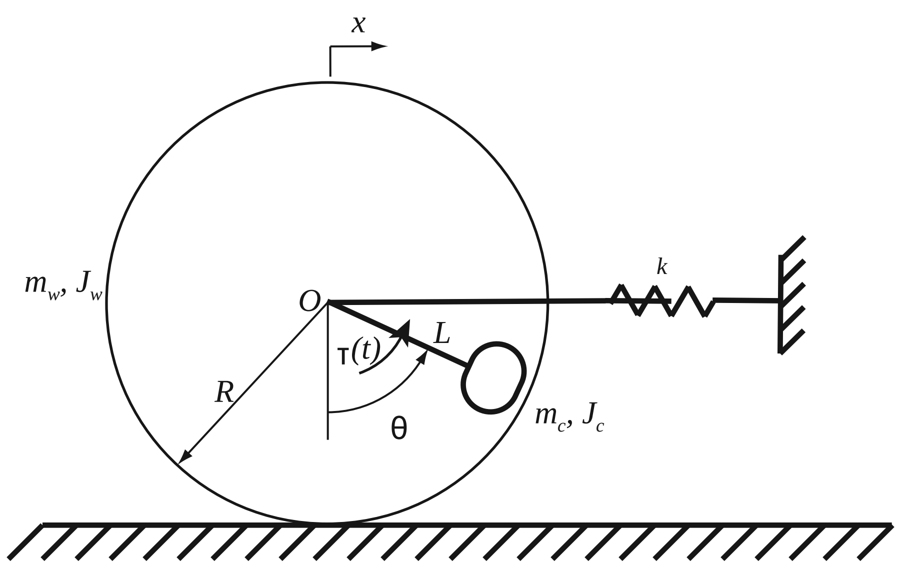
Three masses are attached to a stepped pulley by inextensible cables, as shown in the left figure. The pulley meshes with gear 1, of inertia I_1 = 0.15 kg·m^2 and radius r_1 = 100 mm, and gear 2, of inertia I_2 = 0.25 kg·m^2 and radius r_2 = 200 mm. The masses of the blocks are m_A = 2 kg, m_B = 1.5 kg, and m_C = 1.2 kg. The radii of the pulley are r_A = 60 mm, r_B = 210 mm, and r_C = 350 mm.

a. At a certain instant, the pulley rotates with a clockwise angular acceleration of 6 rad/s^2. What is the mass moment of inertia of the pulley about point *O*, the center of the pulley?
b. If you were to replace the system with a single equivalent inertia I_e and a single gravitational moment M_g (as shown in the right figure), what would the equivalent inertia and moment be?





As a newly minted Ph.D. graduate, you have been asked to help analyze the following system in your new company:

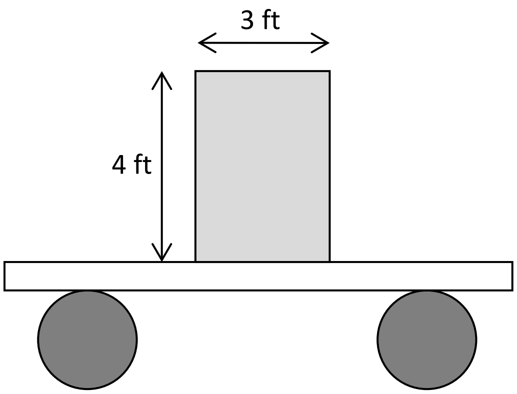


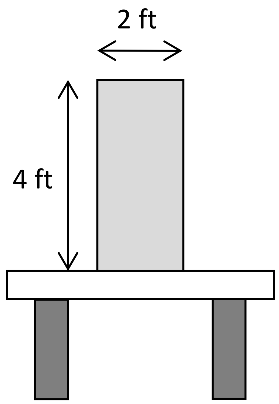
This is a theme park ride where riders will sit in a cage with a total mass m_c and a mass moment of inertia J_c. The cage will be spun around a pivot point O by applying a torque $\tau(t)$. The rod connecting the cage to the pivot point has length L and negligible mass. The pivot point O is in the center of a large wheel with radius R, where R is greater than L to allow the cage to spin completely around inside the wheel. Linear rotational damping with damping coefficient b acts at the pivot point O between the rod and the wheel. The wheel rolls without slipping on a horizontal surface, and it is attached by a spring with spring constant k to a wall. The wheel has a mass m_w and mass moment of inertia J_w. As the cage spins inside the wheel, this will create dynamic forces that will cause the wheel to roll back and forth on the horizontal surface. Note that the torque $\tau(t)$ acts between the wheel and the pendulum rod; this means that when the wheel and cage are considered separately, the torque $\tau(t)$ will create an equal and opposite torque acting on the wheel.

Do the following:

a) Derive the equations of motion for this system choosing a minimum set of state variables (you should only have equations in terms of x, θ and their 1st and 2nd derivatives. You are not required to put the equations in state variable form.
b) Explain what you would have to do differently if the outer wheel was assumed to be able to slip. Don't find these equations of motion, but do explain what would change.

A 3-by-2-by-4 foot crate weighing 200 lb is on a train car travelling toward a curve with a radius of curvature of 300 ft. The train driver wants to set the cruise control to a constant speed. If the static coefficient of friction between the crate and the train car is 0.6, what is the fastest speed the train driver can set without causing the crate to tip or slide in the curve?





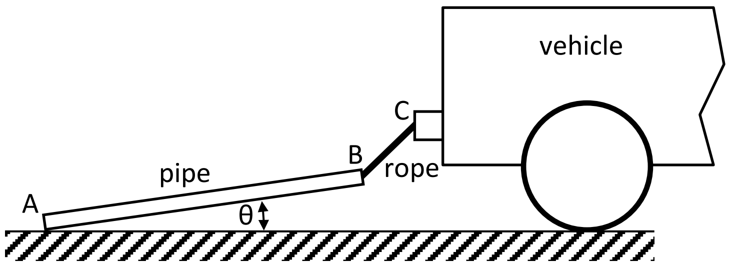
A pipe is dragged behind a vehicle. The pipe is attached to the vehicle by a rope. The coefficient of kinetic friction between the pipe and the street (at *A*) is $\mu = 0.3$. The length of the pipe (from *A* to *B*) is 2.5m, the length of the rope (from *B* to *C*) is 0.5m, and the height of the hitch (*C*) above the road is 0.8m. The mass of the pipe is 400kg.

a) If the angle between the pipe and the street is constant at $\theta = 12°$, what is the acceleration of the car?

The next two questions may be answered qualitatively:

b) When the vehicle is driving at a constant speed, the angle between the pipe and the street is θ_{ss}. Do you expect θ_{ss} to be greater or smaller than 12°? Why? If you were not able to answer the previous question, assume that the answer to the previous question was a positive acceleration (positive to the right).

c) Does θ_{ss} depend on the magnitude of the constant speed? Why or why not?



The 2 kg sphere is projected horizontally with a velocity of 10 m/s against the 10 kg carriage that is backed up by a spring with a stiffness of 1600 N/m. The carriage is initially at rest with the spring uncompressed. If the coefficient of restitution between the sphere and the carriage is 0.6, calculate the rebound velocity v', the rebound angle θ, and the maximum travel δ of the carriage after impact. There is no friction between the sphere and the carriage.