Solid Mechanics PhD Qualifying Exam
Information Sheet and Instructions

Objective

Demonstrate a post-undergraduate level understanding of the exam topics and learning outcomes defined below. Demonstration of these skills includes the ability to apply fundamental principles, make and use appropriate assumptions, and draw connections between concepts. This exam is meant to assess the PhD Program Outcomes of *subject mastery* and *independent learning.*

A student should develop competence in the material covered in an undergraduate mechanics of materials course and an undergraduate machine design course (e.g., CE 203 and ME 372 at BYU). Typical engineering questions in these undergraduate courses range from basic concepts and straightforward application to advanced questions involving complex machines and the synthesis of more than one concept. A student is prepared for the qualifying exam when they have mastered the advanced problems.

Exam Instructions

- The exam will have approximately 3-5 problems not necessarily of equal weight. All problems will be graded. Correct work must be shown on each problem to receive credit (i.e., just an answer is insufficient).
- A score of 70% or higher is considered a passing grade.
- The exam is open book and open notes with the exception of solution manuals, which are not allowed.
- Calculators are required; however, cell phones and other electronic devices are not permitted in the exam room.
- The exam has a time limit of 2.0 hours.
- All work should be in neat engineering style with assumptions clearly stated. Work that is messy or hard to follow will not be graded.

Exam Topics and Learning Objectives

1. **Combined Loads.** Design and analyze machine components subjected to axial forces, bending moments, shear forces, and torsional loads. Select and use appropriate static failure theories for ductile and brittle materials such as Maximum Shear Stress, Distortional Energy, and Mohr-Coulomb. Draw shear moment diagrams and locate the maximum and minimum stresses in machine components. Determine the stresses at a particular point on the surface or throughout the cross section of a machine component. Compute principal stresses and strains and the orientation of the stresses and strains at a point in a machine component. Account for stress concentration factors in ductile and brittle materials.

2. **Deflection.** Estimate the deflection of structural members under combined loading using an energy approach (e.g., Castigliano). Compute reaction forces and moments as well as deflections in statically indeterminate machine components.

3. **Buckling.** Select appropriate buckling failure theories, determine critical buckling loads, and size machine components to prevent buckling failure for statically loaded compression members.

4. **Pressure Vessels and Press Fits.** Design and compute the stresses and strains in axisymmetric machine components. Design and analyze stresses, strains and deflection in press fits.

5. **Fatigue.** Design machine components subjected to fluctuating loads using appropriate fatigue failure theories. Compute the stresses, fatigue life, and predict safety factors in machine members with infinite and finite life.

6. **Beams.** Design and analyze curved beams with built-up cross sections and beams with non-symmetric cross sections.

7. **Stress Concentration Factors**. Design and analyze machine components with stress concentrations made from ductile and brittle materials with static and fluctuating loads.

8. **Bolts**. Size bolts for machine components subjected to static loads. Design and analyze bolted joints.

Representative courses at BYU:	CCE 203: Mechanics of Materials MeEn 372: Mechanical System Design
Representative texts:	Scott, *Mechanics of Materials* Shigley, *Mechanical Engineering Design*

SAMPLE PROBLEMS

Problem 1

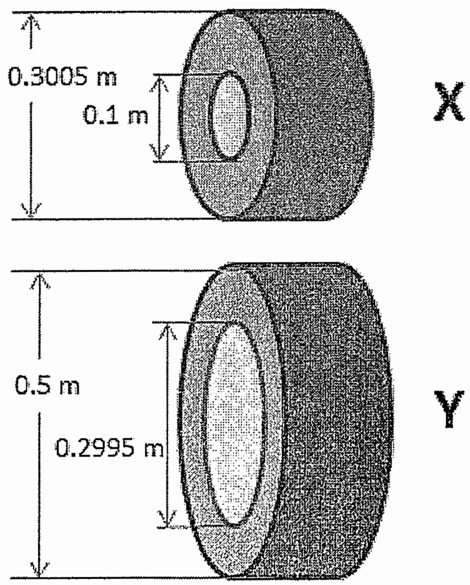
Two hollow cylindrical parts of the same length (shown below) are to be assembled together. Part X is an aluminum alloy with a Young's modulus of 10 Mpsi, a Poisson's ratio of 0.33, and a coefficient of thermal expansion of 13.3×10^{-6} °F^{-1}. Part Y is a brass alloy with a Young's modulus of 16 MPsi, a Poisson's ratio of 0.36, and a coefficient of thermal expansion of 10.4×10^{-6} °F^{-1}. Part X is cooled while Part Y is heated such that they can be assembled. The assembly is then cooled to room temperature.

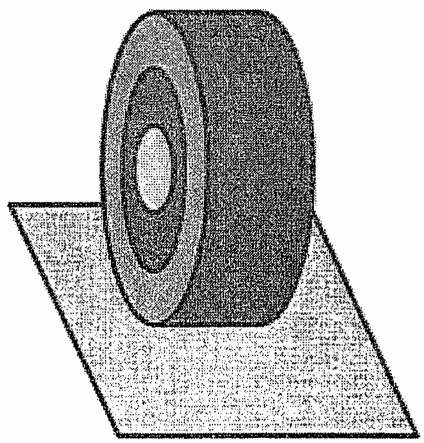
A) **20 Points**. If the temperature of Part X is dropped by 100 °F, determine the minimum temperature that Part Y must be raised such that the 2 parts can be assembled without interference.

B) **40 Points**. Determine the interface pressure (radial stress) after the assembly has been cooled to room temperature.

C) **20 Points**. Determine the change in diameter in each part at the interface of the assembly.

D) **20 Points**. Determine the radial and circumferential stresses induced in each part due to the assembly.





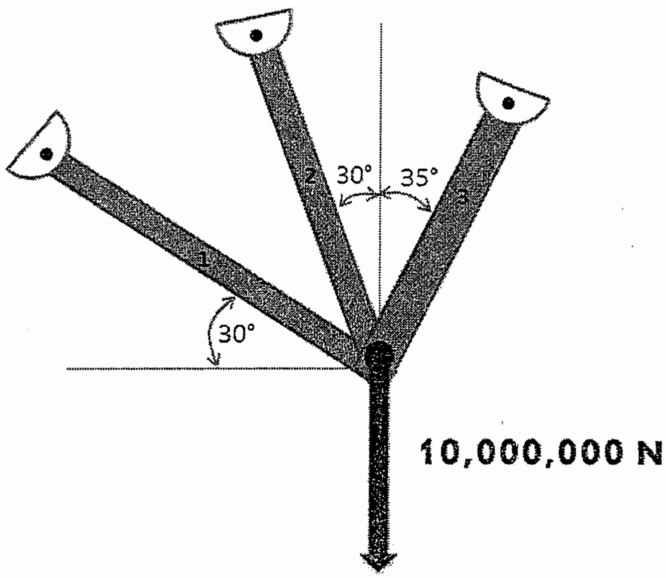
Problem 2

A load is supported by three pinned tension members as shown below. The Young's modulus, length, and cross-sectional areas of each member are given in the table below.

A) **50 Points**. Compute the force in each member.

B) **30 Points**. Compute the strain in each member.

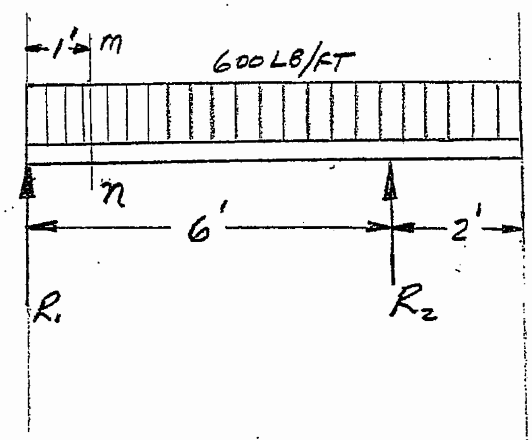
C) **20 Points**. Compute the stress in each member.

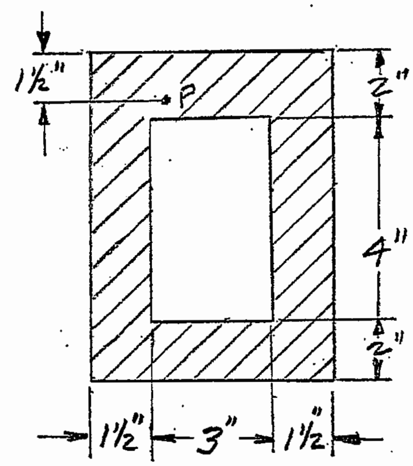


	Link 1	Link 2	Link 3
Young's Modulus	100 GPa	50 GPa	25 GPa
Length	5 m	4 m	3 m
Cross-sectional area	0.050 m²	0.050 m²	0.267 m²

Problem 3

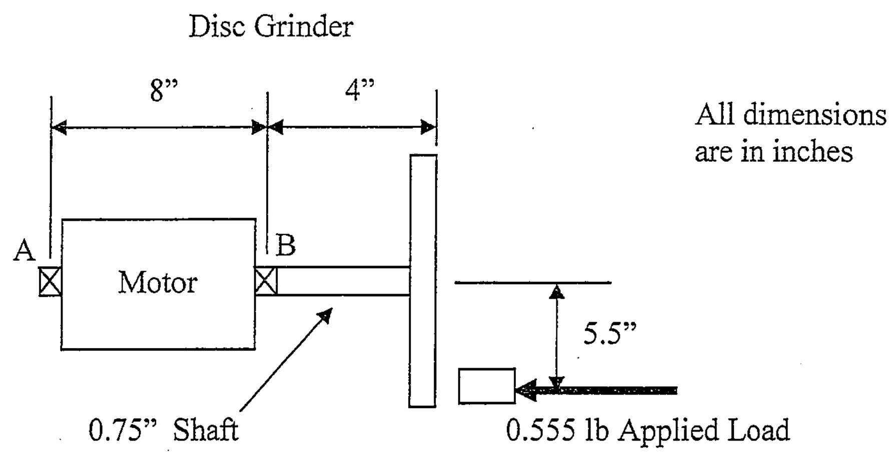
Calculate the maximum shearing stress for the loading applied to the hollow rectangular beam shown in the figure below. Also, find the shearing stress in section mn at points 1 ½ inches below the top face of the beam, shown as point p in the cross section of the beam. Please show all of your work.





Problem 4

The disk sander has an applied load of 0.555 lb as shown. There is a coefficient of friction between the block and the disk grinder of 2.0. Bearing A is able to take both thrust and radial loads. Bearing B is only able to take radial loads. The disk sander shaft is 0.75 inches in diameter. The full-load speed of the motor is 3440 rpm and is powered by a 1/3-hp motor. Find the bearing reactions at A and B. Determine the magnitude and location of the maximum principal stress in the shaft



Strength of Materials PhD Qualifying Exam Fall 2010

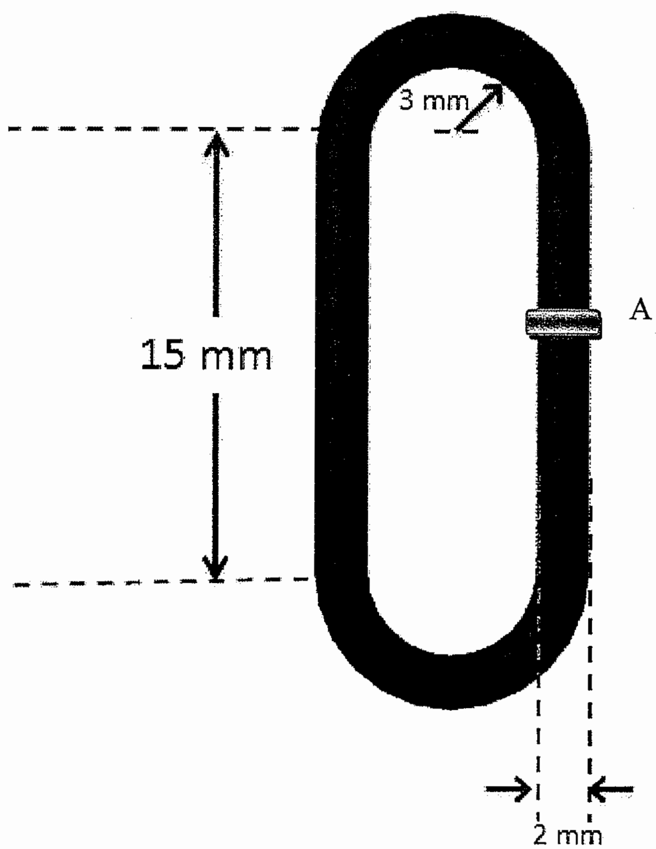
Problem 1

A swingset chain is constructed of welded steel links (σ_{yield} = 250 MPa) with circular cross section in the geometry shown. Assume that the weight of a person sitting on the swing is equally distributed between the two support chains.

A) **40 Points**. Determine the weight (W) that will initiate yield in the chain under static conditions (i.e., the person is sitting still).

B) **40 Points**. Repeat the analysis for the case where the link was not welded (the link is open at point A).

C) **20 Points**. Discuss what factors in the analysis would be affected if the passenger were swinging back and forth with a constant period (pendulum motion). What additional information would be required to perform the analysis? Describe how you would solve the problem.



Problem 2

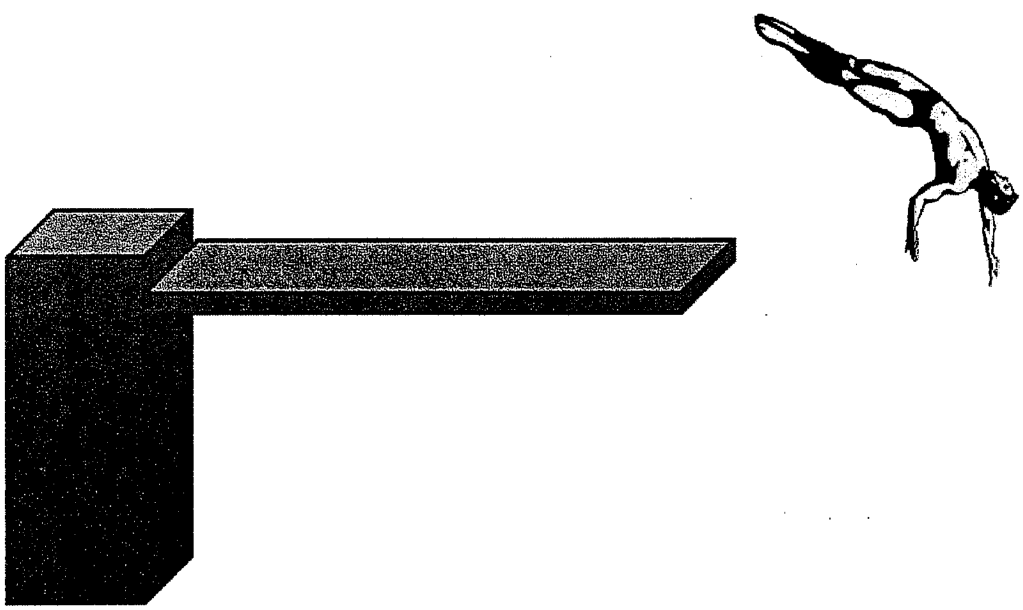
An aluminum diving board may be assumed to act like a cantilever beam subjected to an end load. The diving board is 6 ft long, 1 inch tall, and 12 inches wide. The aluminum used in the diving board has an ultimate strength (S_u) of 40,000 psi, a yield strength (S_y) of 34,000 psi, and an endurance limit (S_e) of 16,000 psi. The modulus of elasticity is 10,300,000 psi.

A) **30 Points**. Compute the safety factor versus yielding in the diving board under a static 400 lb end load.

B) **30 Points**. If a 200 lb end load is applied to the diving board as a fully-reversed cyclic load with a frequency of 0.5 Hz and a mean stress of 4,000 psi, use the modified Goodman relation to determine the safety factor of the diving board.

C) **30 Points**. A 2 inch diameter hole is drilled through the middle of the diving board. Compute the safety factor of the diving board under the same conditions described in B).

D) **10 Points**. Describe how you could use Miner's rule to estimate the endurance limit of the diving board after it was exposed to a known number of bounces from divers of various weights, ranging from 100 lb to 500 lb.



Problem 3

An aluminum alloy column is fixed at one end and round at the other. The column is 5 ft. long and is actually a plate 4 in. wide and t in. thick.

What is t if the working load on the column is desired to be 20,000 lbs and a factor of safety against buckling is desired to be 3.0?

In addition to determining t, the thickness of the plate, explain and justify your method of analysis.

Problem 4

A pressure vessel 5 ft. in diameter is made up of solid steel plate 0.5 in. thick. The internal pressure in the vessel is 200psi. At a point on the exterior surface of the cylindrical portion of the vessel, find the following:

a.) The maximum shear stress in any plane perpendicular to the plane determined by the principal stresses

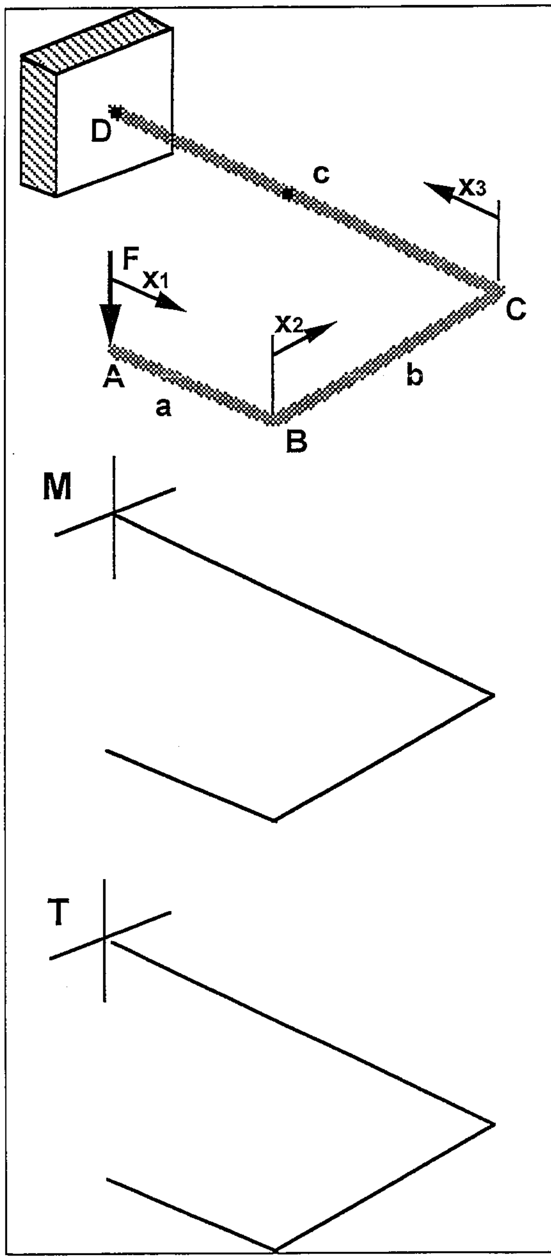
b.) The maximum shear stress at the point in any plane

c.) At a point on the interior surface of the vessel, find the maximum shear stress in a plane perpendicular to the plane containing the circumferential stress and the pressure.

d.) Draw a diagram showing the planes of pure shear

PhD Qualifying Exam – W 2010
Strength of Materials

Problem 1



The bracket **ABCD** is formed in a horizontal plane and cantilevered at the wall (point **D**), as shown. A vertical force **F** is applied at point **A**.

1. Sketch the bending moment diagram and torque diagram for segments **a, b,& c** on the coordinates beneath the diagram, or use another sheet of paper.

2. Write equations for the moments and torque for each segment in terms of **F** and the **x1, x2, & x3** coordinates.

3. Derive an expression for the vertical deflection at point A, due to the load **F**, in terms of the lengths **a. b. & c**, the force **F** and the bending properties **EI** and torsion properties **GJ**.

4. What is the equivalent stiffness for the bracket for loads applied at point A.

   Assume a uniform cross section.
   Neglect the rounded corners.
   Negative deflection is vertically down.
   State any other assumptions.

   Use any method you wish, ie.
   Superposition, Castigliano, etc.

Problem 2

A 2 leg wooden "ladder" ($E = 2 \times 10^6$ psi, $S_y = S_{ult} = 7.3 \times 10^3$ psi) is loaded as shown below (P is a purely vertical load). The legs are attached to the ground such that they can freely rotate, but will not slide (a pinned end condition). The top plate of the ladder is rigidly attached (won't rotate or slide at leg attachments). Ignore the weight of the ladder.

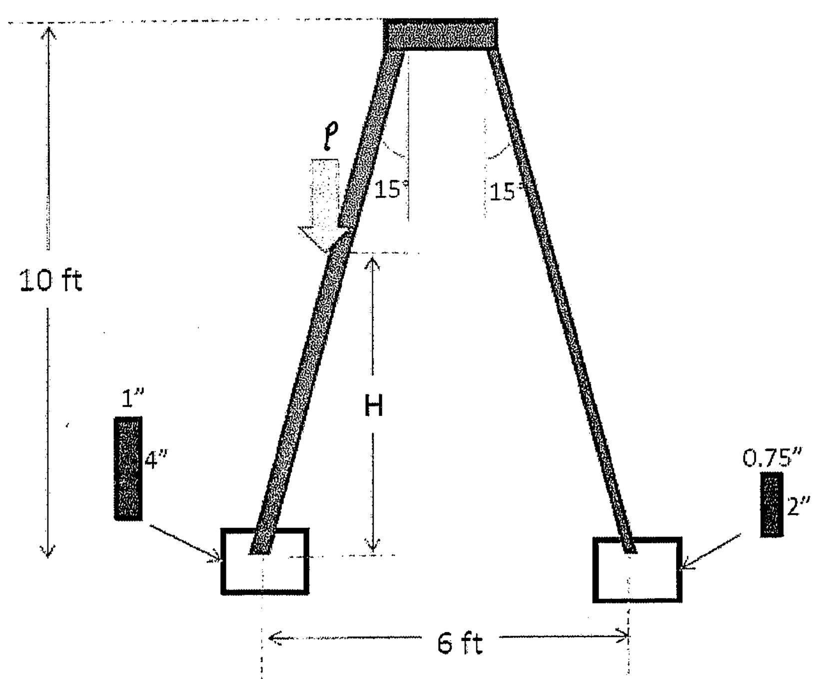
A) **10 Points.** Develop a free-body diagram of the loaded ladder.

B) **20 Points.** Compute the end-reactions at each of the legs as a function of the applied load (P) and the height (H).

C) **25 Points.** Compute the critical column buckling load for each leg in terms of H. Use the radius of gyration to determine whether each leg qualifies as an intermediate column or a long column.

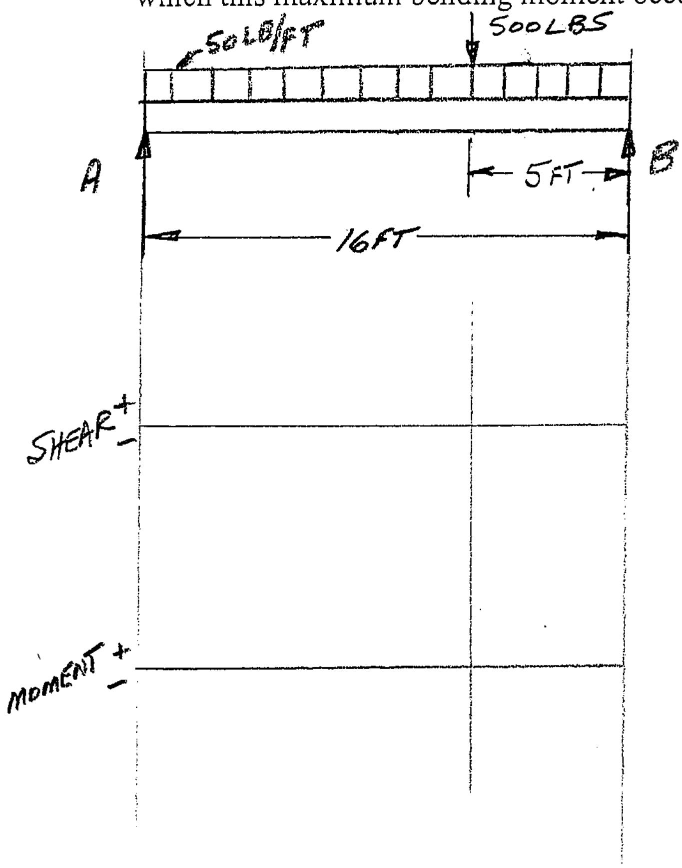
D) **25 Points.** If the height (H) of the applied load is 8 feet, how much load (P) can be applied before buckling occurs? In which leg will it occur? Assume that the lateral component of the load does not affect the buckling load.

E) **20 Points.** Discuss how you would include the influence of the lateral loading component in your calculations (hint: the lateral load will induce a curvature in the leg of the ladder).

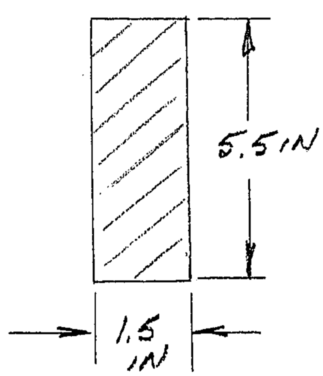


Problem 3.

For the beam loaded as shown, sketch the shear and bending moment diagrams and determine the maximum bending moment that exists in the beam and the location at which this maximum bending moment occurs.



Assuming that the beam has a rectangular cross section as shown, calculate the maximum tensile stress in the beam.



Brigham Young University

Name________________________________

(Open book, notes)

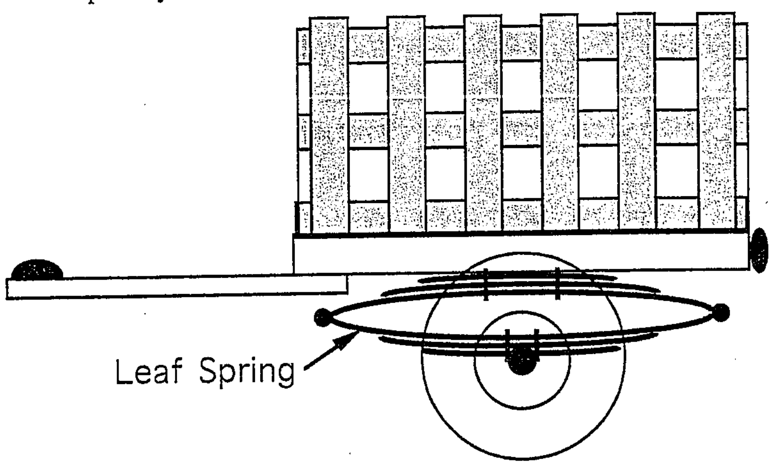
PhD Qualifier

(Show all formulas and calculations)

(100) Problem 4

A small utility trailer is suspended on a pair of double leaf springs mounted over each tire, as shown. The bottom half of each spring is bolted to the trailer axle. The top half is bolted to the underside of the trailer bed.

Each beam behaves like a simply supported beam, loaded at the center. The multiple layers of the spring are designed to strengthen the mid-section, where the bending moment is greatest. Question. Why the multiple layers instead of a solid beam of the same thickness?



Specifications:

Material: Hot-rolled 1050 carbon steel Q & T at 800°F.

S_{ut} = _158Kpsi________ S_{yt} = _115 Kpsi________

Cross Section: (For calculating the size factor.)

Each leaf is 2.0 in. wide by 0.1875 in thick.

The springs are clamped by means of U-bolts. Let $K_f = 1.2$

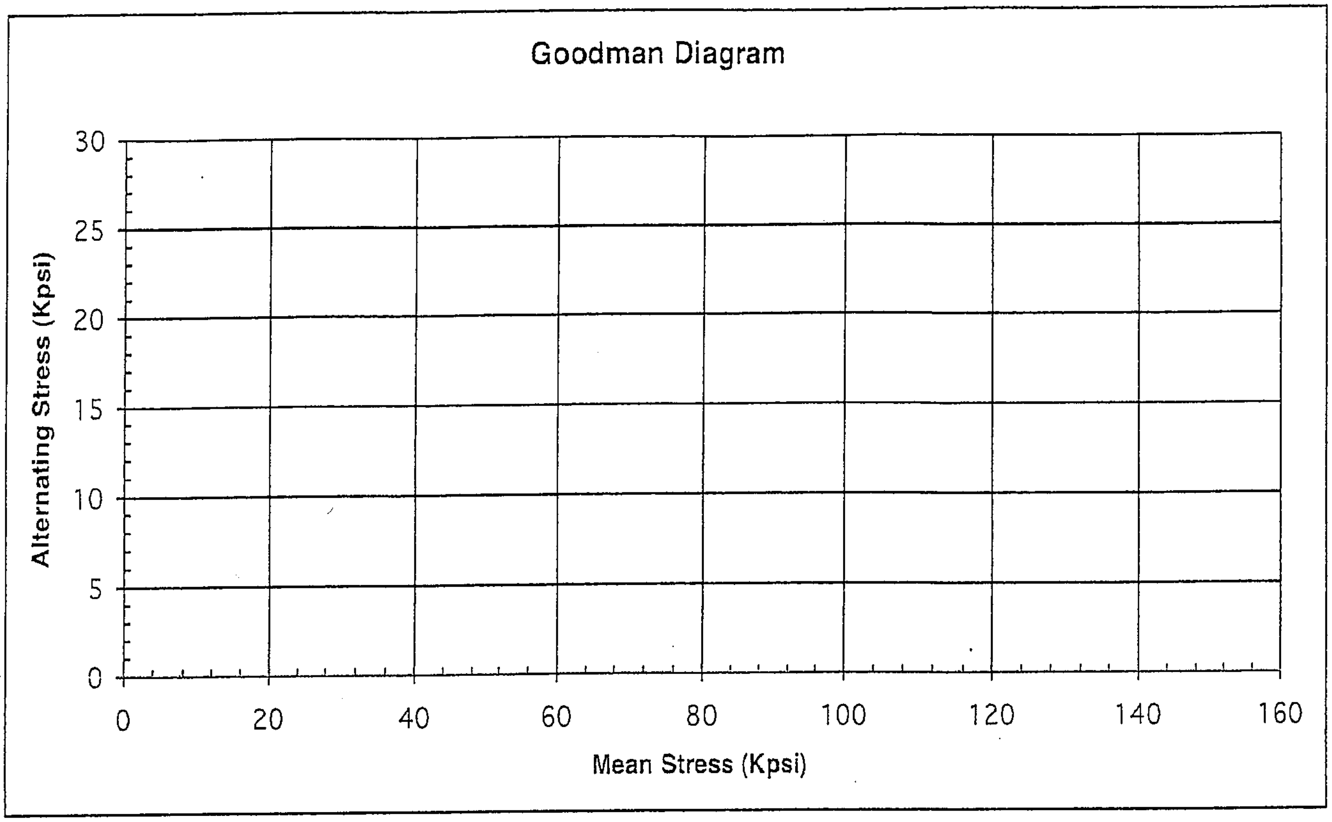
Loads: The combined weight of the trailer and load is 800 lbs., or 400 lbs on each spring. This produces a static bending stress of 45,000 psi at the midspan.

a. Compute the endurance limit for the spring. A reliability of 99.9% is desired.

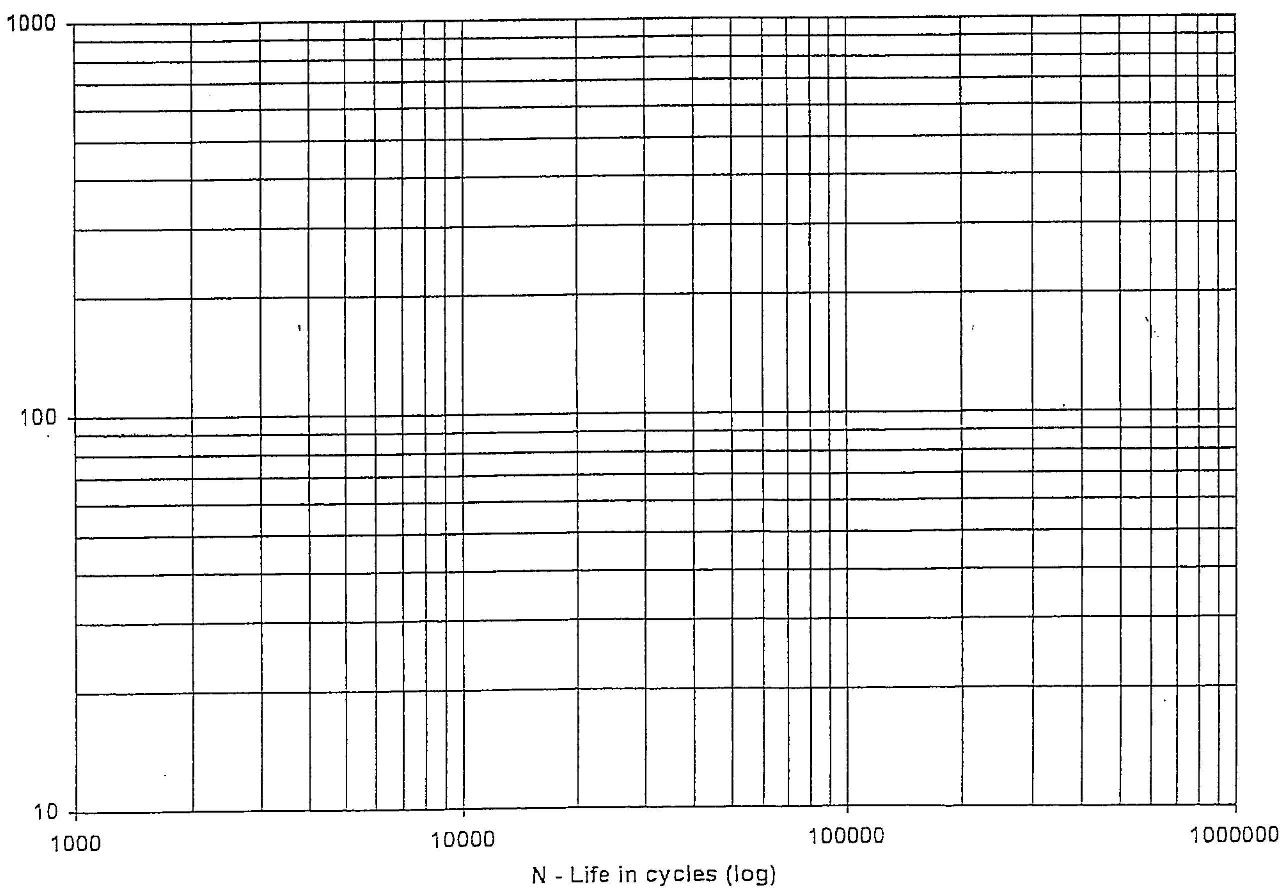
b. **Case 1.** Pulling the trailer on the highway excites vibrations, creating an alternating stress which is superimposed on the static stress. Estimate the value of σ_a which may be applied cyclicly (at constant σ_m) for an infinite life and a safety factor of 2.0. Show all the necessary diagrams. Solve analytically or graphically (to a good scale).

c. **Case 2.** Suppose an unpaved road produces an alternating stress of 20,000 psi. with the same 800 lbs. static load. Estimate the life in cycles before the springs might fail by fatigue. Show with appropriate diagrams how you obtained this estimate.

d. **Case 3.** For a constant alternating stress of 7500 psi., how much can the static load (lbs.) be increased before fatigue failure would be predicted? Show calculations, load lines, etc.


Goodman Diagram
Alternating Stress (Kpsi)
30
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
Mean Stress (Kpsi)

S-N Diagram


1000
100
10
1000
10000
100000
1000000
N - Life in cycles (log)

Problem 1
(Please show all formulas, assumptions and calculations. Open book and notes)

As a consultant you have been asked to investigate the safety of an existing, seamless walled, pressure vessel made of type 304 stainless steel. The wall thickness of the vessel is 0.100 inches, and the vessel has an outer diameter of 10 inches. The vessel is pressurized with water at 1500 psi. The vessel is 15 inches in length and has hemispherical end caps which have been welded to the cylinder and are twice the thickness of the cylinder's wall.

a.) 20 pts. Calculate the maximum circumferential and longitudinal stresses in the circumferential wall of the pressure vessel.

b.) 30 pts. What are the maximum tensile and/or compressive principal stresses that exist in the cylinder wall?

c.) 20 pts. What is the maximum shear stress that exists in the cylinder wall?

d.) 20 pts. Assuming the yield strength of type 304 stainless steel is 30,000 psi. What is the factor of safety related to the yield strength of the cylinder wall material?

e.) 10 pts. Briefly discuss what other aspects of the cylinder's design and/or manufacturing might be of concern to you, given the working pressure currently being used in the cylinder.

Problem 2

The bracket shown is attached to a column with three 0.75 in bolts arranged in an equilateral triangular layout. A force is applied 24 inches from the center of the bolt group.

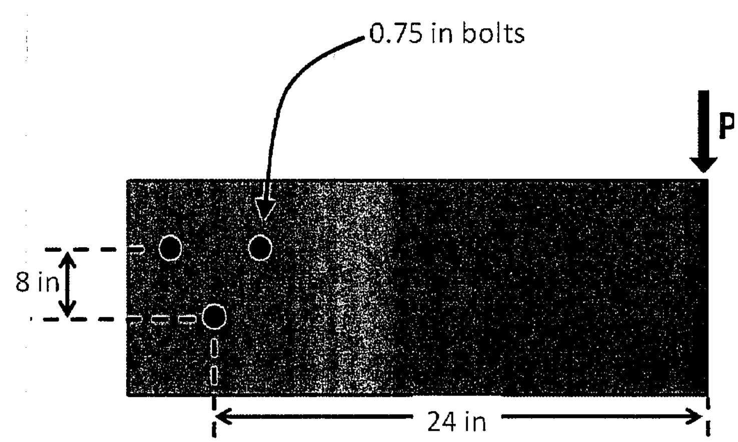
A) **20 Points.** Compute the vertical shear load on each bolt due to the applied load (P).

B) **30 Points.** Compute the shear load on each bolt due to torque.

C) **15 Points.** What is the maximum stress seen in the most highly stressed bolt due to the applied load (P)?

D) **15 Points.** If P is applied as a sinusoidal load varying in magnitude between 0 lb and 1000 lb, what are the magnitudes of the mean and alternating stress seen at the most highly stressed bolt?

E) **20 Points.** Discuss the effects of preload and joint friction on this bolted connection.



Brigham Young University

Name________________________
(Open book, notes)

Strength of Materials
(Show all formulas and calculations)

(100) Problem 3

A front wheel spindel of an automobile is constructed of forged ANSI 1045 steel (S_y = 104 ksi, S_{ut} = 145 ksi). Assume the load on the wheel may be represented by a fluctuating force as shown, having the following range:

F_{min} = 1000 lbs

F_{max} = 2000 lbs

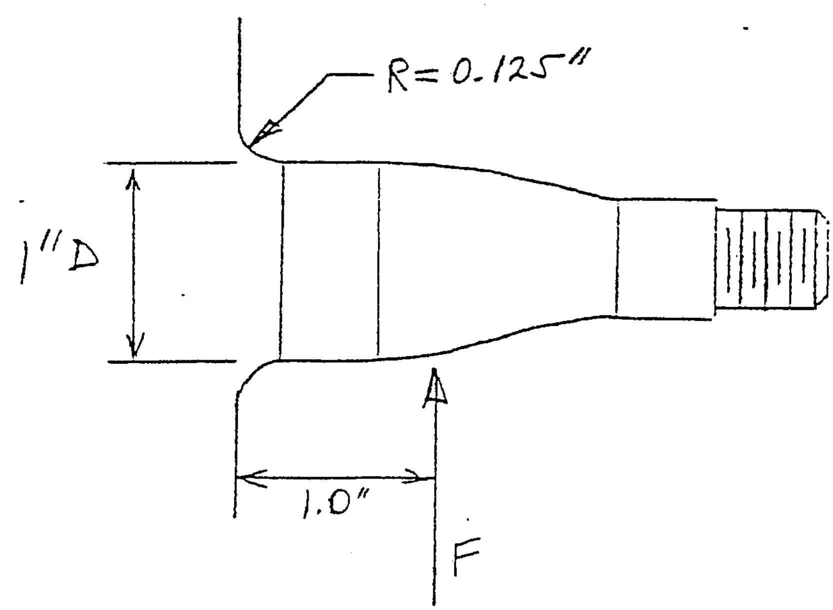
Note: A spindel is a non-rotating shaft.
Mating surfaces are ground, but the fillet surface is as-forged.

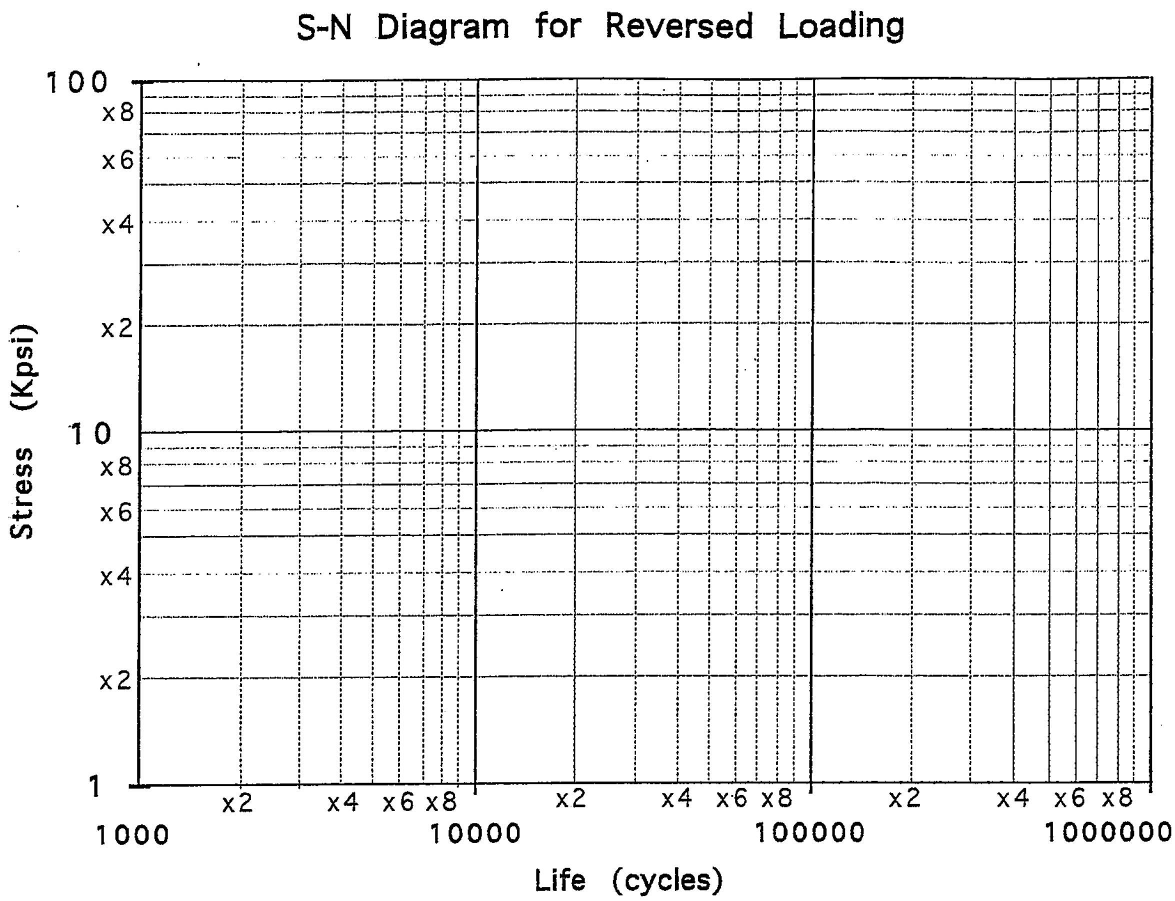
a) Calculate σ_a, σ_m.and fatigue stress concentration factor K_f.

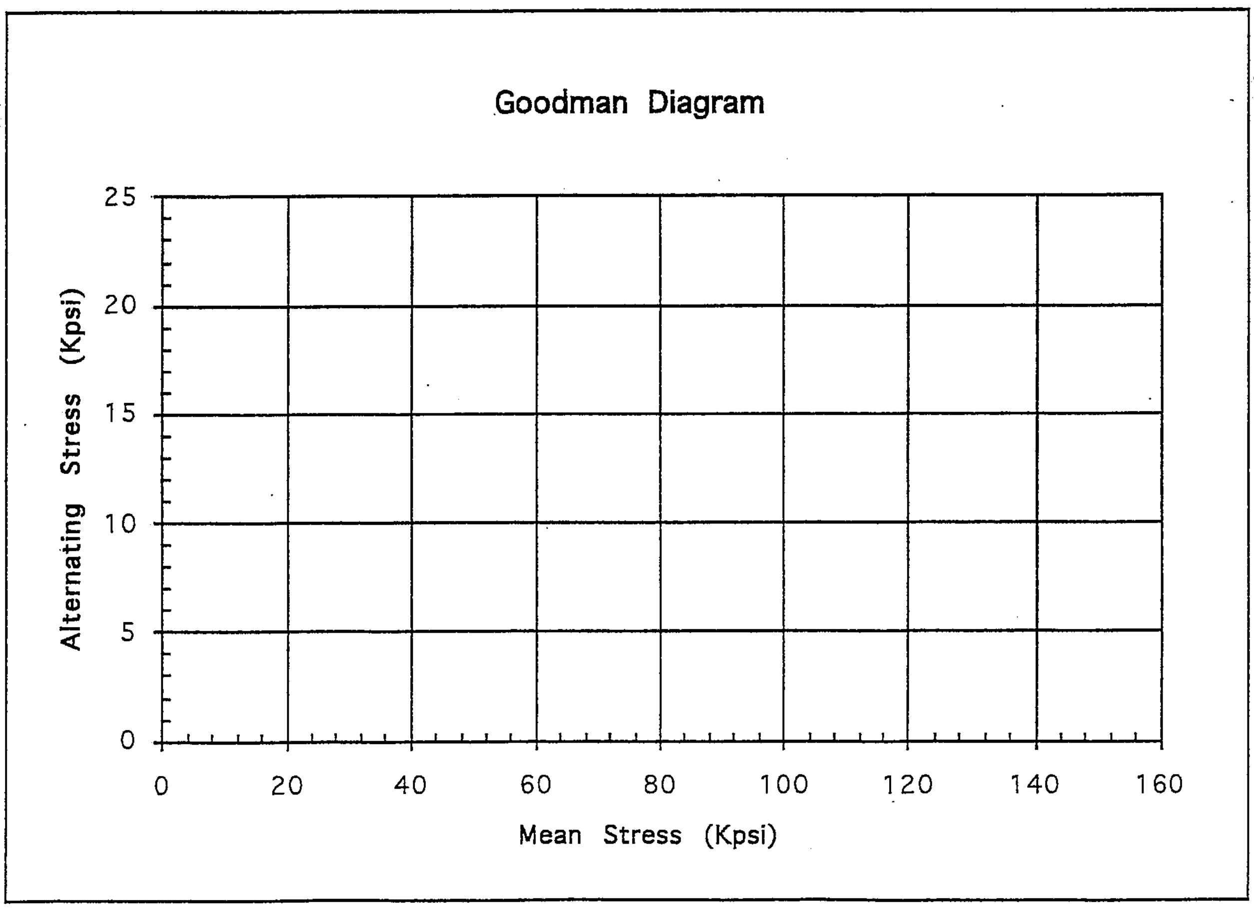
b) Calculate Se for 99% reliability.

c) Draw the appropriate fatigue strength diagram.

d) Assuming F_{mean} is constant, what is the safety factor for infinite life? (Compute graphically or analytically).




S-N Diagram for Reversed Loading
100
x8
x6
x4
x2
10
x8
x6
x4
x2
1
Stress (Kpsi)
x2
x4
x6
x8
1000
10000
100000
1000000
Life (cycles)


Goodman Diagram
25
20
15
10
5
0
Alternating Stress (Kpsi)
0
20
40
60
80
100
120
140
160
Mean Stress (Kpsi)

Brigham Young University
PhD Qualifier

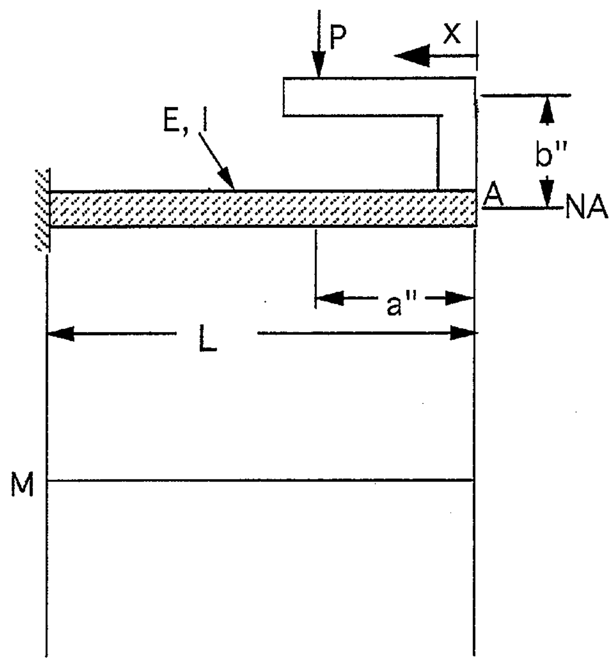
Name______________________
(Open book, notes)

(Show all formulas and calculations)

(100) Problem 4

A uniform beam is loaded by means of an angle bracket attached rigidly at end A, as shown, which transmits loads to point A. The beam cross section is rectangular, with properties E and I.

> Hint: Replace the angle bracket with an equivalent load and moment at point A on the beam. You would be wise to treat them as independent loads until you have derived the deflection expressions.



a. Find the reactions at the wall.

b. Sketch the bending moment (M) in terms of loads P and M_A throughout the length L. Write equations for M vs. x throughout the length.

c. Using Castigliano's theorem, derive an expression for the vertical deflection at point A in terms of P, L, **a** and EI. (Or, use superposition of the loads at A.)

d. Repeat for the rotation at A.

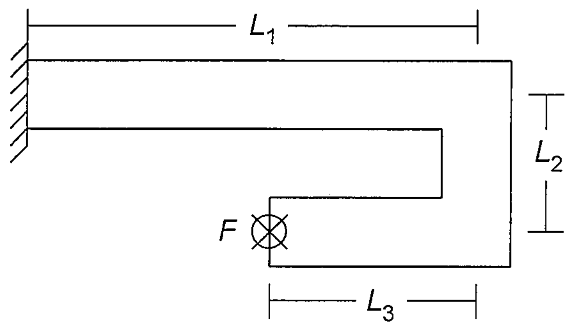
e. How much would you have to increase the bracket length **a** to produce zero vertical deflection at end A? (As a fraction of L)

f. What value of length **a** would produce zero rotation? (As a fraction of L)

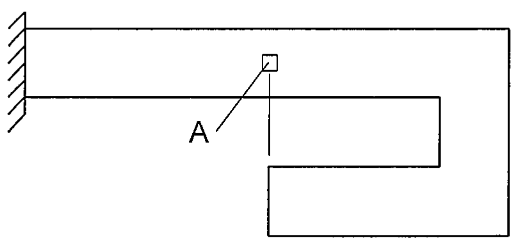
g. How does the deflection in the bracket affect the problem?

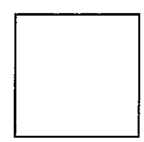
Problem 1

Consider the part shown in the figure. The three segments each have a circular cross section with diameter D. A force, F, is acting downward (into the page) at the position shown.



(a) Calculate the stresses at point A (assume it is on the top surface) shown in the image below. Sketch the stresses on the stress element (such as the one shown to the right), and state the stresses symbolically. (The stresses should be written using the given parameters of F, D, L_1, L_2, L_3, and π.)





(b) Where will the maximum stress occur? Write symbolically the values of the stresses on a stress element at that location. (The stresses should be written using the given parameters of F, D, L_1, L_2, L_3, and π.)

Problem 2

A bar is 30 cm long and has a rectangular cross section with height 3 cm and width 4 cm. It is fixed at one end and free at the other. It is made of 1095 HR steel with yield strength of 455 MPa, an ultimate tensile strength of 827 MPa, a Young's modulus of 207 GPa, Poisson's ratio of 0.28, and density of 7.8 Mg/m^3. The bar is curved with a centroidal radius (the distance from the center of curvature to the centroidal axis of the bar) is 10 cm. It is loaded with a moment of 10,000 N-m. The cross section is oriented such that r_o-r_i is the 3 cm dimension.

(a) Calculate the distance from the centroidal axis to neutral axis.

(b) Calculate the bending stress at the inner and outer fibers.

Problem 3

The long and slender beam shown below has a mass dropped on it.

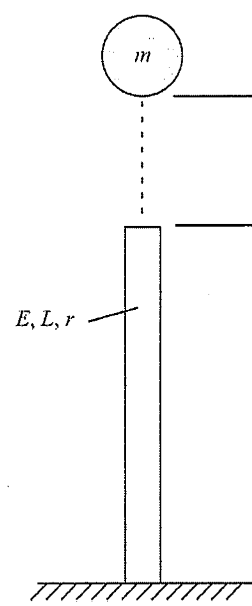
a. Will the impact load cause this beam to buckle?

Use the following data to backup and prove your answer. The beam has a circular cross section defined by radius $r = 0.01$ m. The mass, $m = 100$ kg, is dropped from a height, $h = 0.1$ m, onto the beam directly inline with the beam's longitudinal axis. The beam material is 1020 steel, hot rolled. The beam's length, L, is 0.5 m.

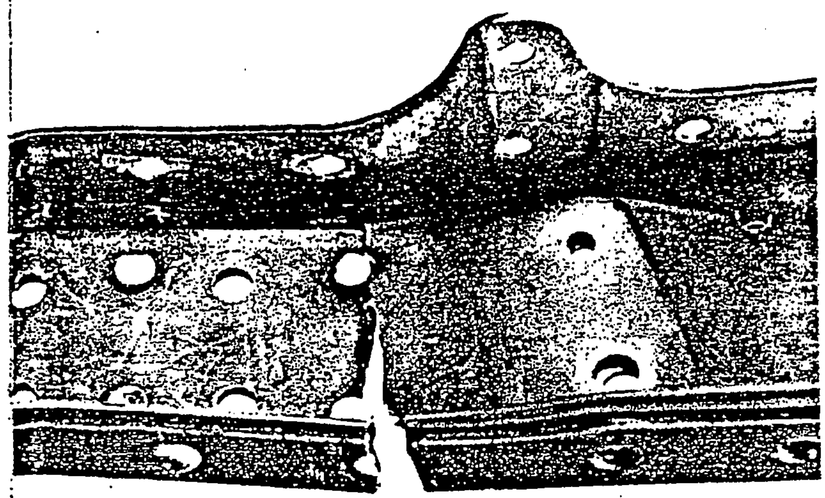
Discussion by Short Response:

b. Discuss and justify your selection of the beam's boundary conditions.

c. Discuss and justify your selection of either the Johnson column failure criteria, or the Euler column failure criteria.



Problem 4



A support member in an automobile suspension system is subjected to repeated bending loads. Several failures have occurred at a particular rivet hole, as shown. Nominal stresses at the rivet hole are:

$\sigma_{max} = 24{,}000$ psi $\quad \sigma_{min} = 4{,}000$ psi

The material is 1030 steel, cold drawn $\quad S_y = 64$ ksi $\quad S_{ut} = 76$ ksi

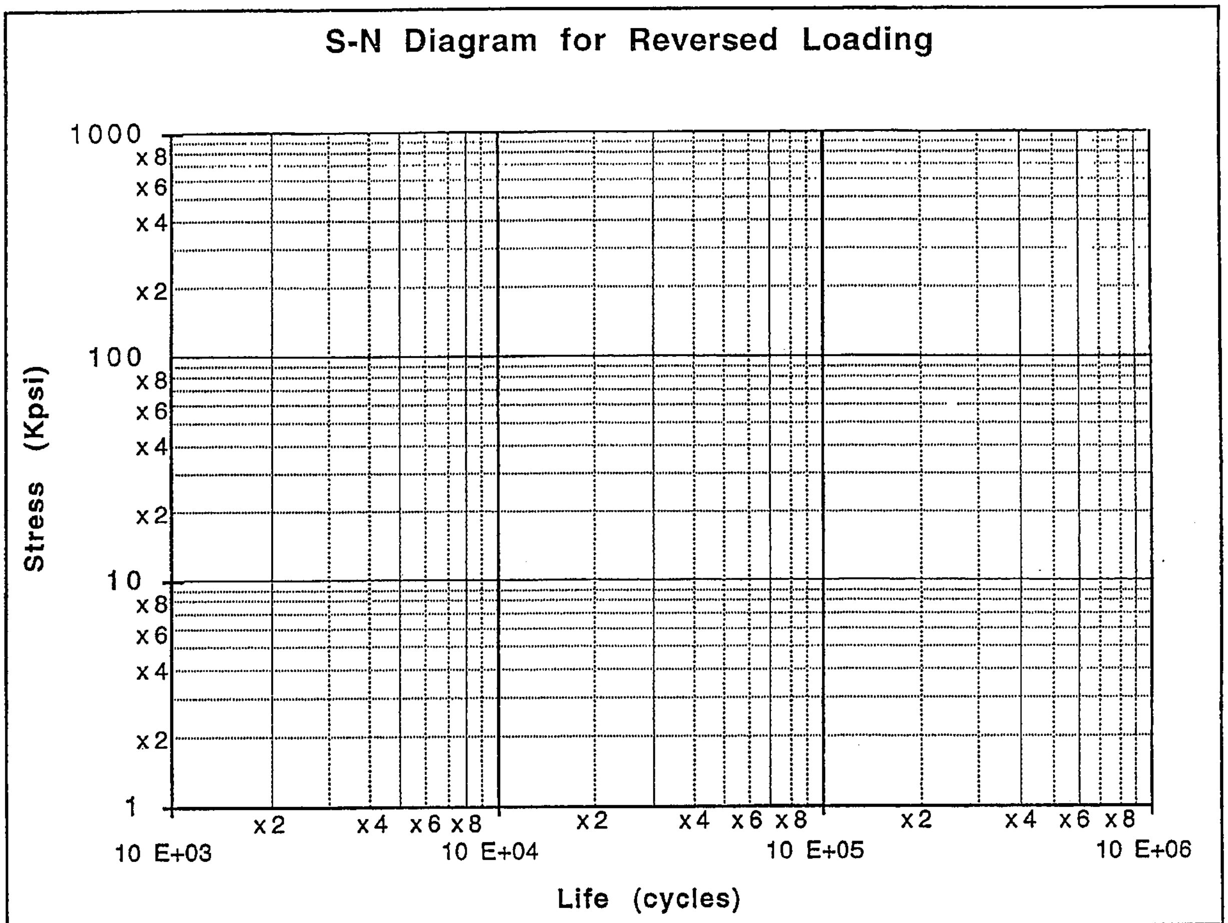
The fully corrected endurance limit is: $\quad S_e = 9.2$ ksi

a) Construct the S-N Diagram and the Goodman Diagram.

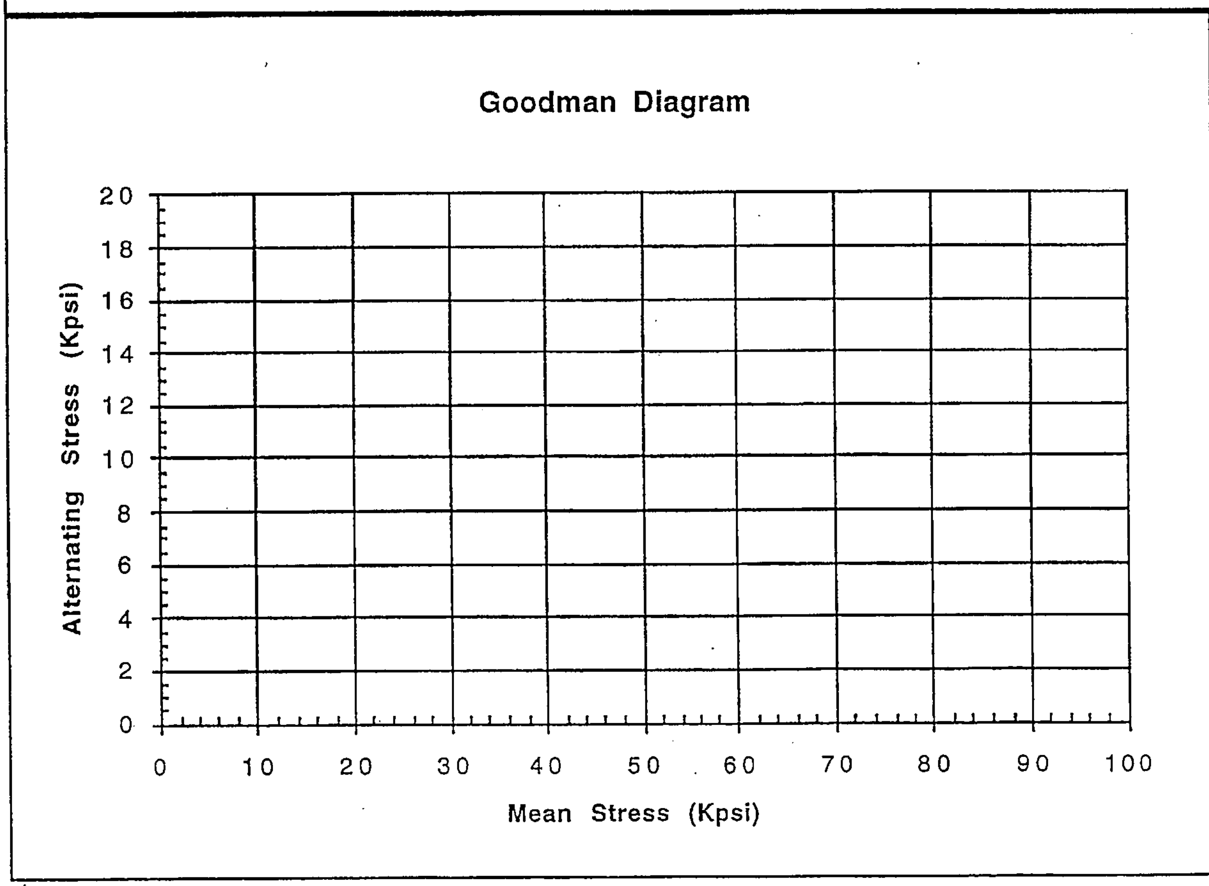
b) Calculate the mean and alternating bending stresses σ_m, σ_a

c) Compute the factor of safety for an infinite life.

d) Compute the factor of safety for a life of $2\text{x}10^5$.



S-N Diagram for Reversed Loading
Stress (Kpsi)
1000
x8
x6
x4
x2
100
10
1
x2
x4
x6
x8
10 E+03
10 E+04
10 E+05
10 E+06
Life (cycles)

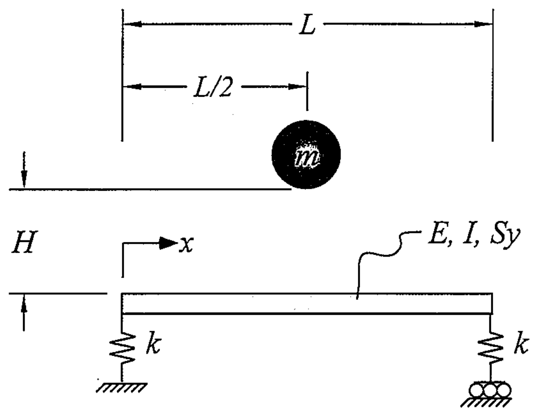


Goodman Diagram
Alternating Stress (Kpsi)
20
18
16
14
12
10
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100
Mean Stress (Kpsi)

Open Book, Open Notes. Show all your work.

(50) Problem 1

A spherical ball of mass m falls from a height H on to a hardwood floor. The system is modeled as shown below. The beam (floor) is considered simply supported. There are however, linear springs at $x = 0$ and $x = L$, which allow the entire simply supported beam to displace vertically. This displacement is governed by the stiffness of the linear springs shown in the figure. Both the beam and the linear springs stay within the linear deflection range.



The following information defines the problem.

Item	**Detail**
Beam Material	*Wood*, E = 6.9 GPa, Sy = 41 MPa
Length of Beam	L = 0.4 m
Width of the Beam	b = 0.041 m
Height of the Beam	h = 0.041 m
Height from which ball is dropped	H = 1.22 m
Spring Stiffness	k =17,500 N/m
Safety Factor	SF = 2
Mass of beam and Mass of spring	Neglect

1. Develop an equation for the maximum mass that can be dropped on the floor as a function of safety factor on the maximum bending stress for the beam. Leave everything symbolic.

2. Substitute the parameter values from the table above into the expression that you developed in part 1. Write the numerical value for the maximum mass.

3. Check your answer by plugging the mass that you obtained from part 2 above into the fundamental relationships for free-fall impact. Find the safety factor. If you solved parts 1 and 2 correctly, your safety factor will be exactly equal to the safety factor shown in the table.

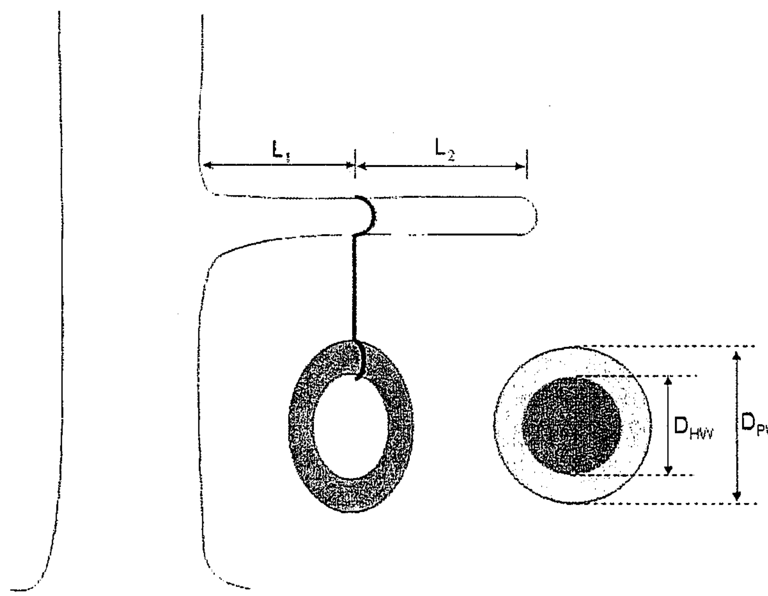
4. Discuss in words and perhaps with pictures how you would change your analysis if the roller condition on the right end of the beam were changed to a fixed condition, so as to be equivalent to that on the left.

(50) Problem 2

A tire swing is hung on the branch of an oak tree as shown. The oak branch may be considered to be a composite beam with a stiff inner portion (the heartwood) and a more compliant outer portion (the pulp wood). Properties for heartwood and pulp wood are given below. The tire swing is attached at a distance (L_1) of 3 meters. The branch extends past the tire swing a distance (L_2) of 4 meters.

Heartwood:
$\rho_{HW} = 850\ kg/m^3$
$E_{HW} = 25$ GPa
$D_{HW} = 10$ cm
Yield Strength = 80 MPa

Pulpwood:
$\rho_{PW} = 1080\ kg/m^3$
$E_{PW} = 13$ GPa
$D_{PW} = 15$ cm
Yield Strength = 50 MPa



a) Determine the maximum static load that can be sustained on the swing at safety factor of 1.5 with respect to yield.

b) What is the deflection at the end of the branch when a 100 kg man stands on the tire swing?

c) If the 100 kg man swings back and forth in a 90 degree arc (-45 degrees to +45 degrees) on the tire swing, write an equation that describes the vertical deflection of the end of the branch as a function of angle.
(Hint – what are the equilibrium equations in the normal and tangential directions?)

(50) Problem 3

Consider the stress element shown in the figure below (kpsi = 1000 psi = 1000 lb/in^2):

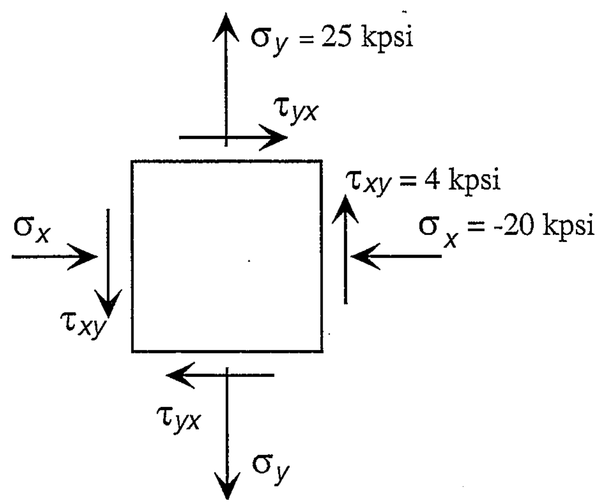
(a) Calculate the principal stresses.

(b) Determine the safety factor for ductile failure if the material is type 301 annealed stainless steel (S_y = 40 kpsi). Provide answers for both maximum shear stress and distortion energy theories.

(c) Did the calculations in part (b) result in the same or different safety factors for the maximum shear stress theory or the the distortion energy theory? If the same, explain why. If different, explain why and state when you would use which theory.

(d) Calculate the safety factor for brittle failure if the material is class 40 gray cast iron (ultimate tensile and compressive strengths of 290 MPa and 965 MPa, respectively).

(e) State the failure theory used in part (d) and explain why it was selected.



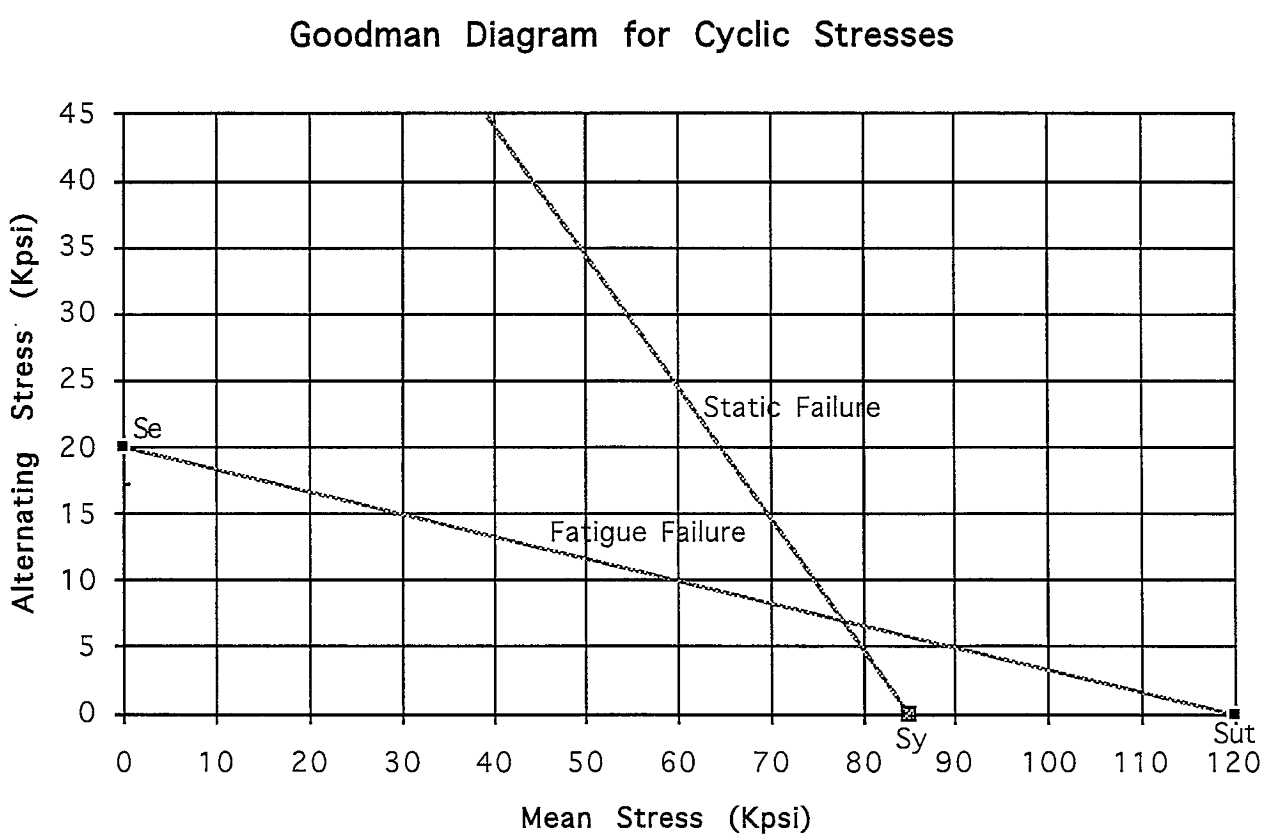
(50) Problem 4

The Goodman diagram shown below was constructed to evaluate the fatigue durability of a machine. The critical part has material properties: $S_y = 84$ kpsi, $S_{ut} = 120$ kpsi, $S_e = 20$ kpsi.

Stress Data: Peak stress $\sigma_{max} = 35{,}000$ psi
Minimum stress $\sigma_{min} = 15{,}000$ psi

1) Complete the Goodman diagram. Plot σ_a, σ_m. Draw the load line, assume proportional loading. Show the limiting stress values on the diagram. Find the safety factor for infinite life.

2) If the alternating stress is held constant, how much can the mean stress increase before failure occurs? Draw the load line. Show the limiting stress value. Calculate the safety factor. How can there be two safety factors for the same σ_a, σ_m.applied loads?



Problem 4 - continued.

3) The S-N diagram shown below represents the fatigue strength for <u>completely reversed cyclic loads</u> for the same material as in (1), on the previous page (S_y = 84 kpsi, S_{ut} = 120 kpsi, S_e = 20 kpsi) . Using the S-N diagram below, find the fatigue strength S_f for a finite life of 200,000 cycles.

4) Demonstrate how to use this result from the S-N diagram to determine the corresponding fatigue strength for the <u>mean-plus-alternating stress</u> values you calculated for the Goodman diagram. That is, calculate the safety factor (or find it graphically on the Goodman diagram) for a finite life of 200,000 cycles. (Assume proportional loading.)