Determining the Optimal Fixed Gear Ratio for an SAE Baja Acceleration Event

Title

(An Example Technical Report)

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Abstract

The acceleration event is one of the scored performance tests in the SAE Baja student competition. The score in this event is based on the time required for a Baja car that is initially at rest to travel 150 ft. Baja cars are required to use 10 hp Briggs and Stratton engines. All of the winning cars have also used a continuously varying transmissions (CVT) with a second fixed gear to achieve easy drivability and overall high gear ratios. This report presents a model and experimental validation for determining the fixed gear ratio that should be used with a given CVT. Based on the model, the optimal fixed gear ratio is 8:1. The measured acceleration time for a Baja Car with a 3.5:0.9 CVT ratio and 8:1 fixed gear ratio was 6.0 ± 0.27 s, which agrees with the 5.8 s predicted by the model. Additional measurements would be required to determine if the model is accurate over a wider range of operating conditions.

Brief Summary of the entire document (IMRAD)

The first three sentence are an **I**ntroduction.

The **M**ethod is briefly described. Both a model and measurements will be used.

Results of the model **and** measurement are **D**iscussed.

A conclusion on the accuracy and suggestion for additional testing is given

Introduction – Literature Review – Objective Statement

Acceleration is an important performance parameter in almost all types of vehicles. The Society of Automotive Engineers (SAE) Baja competition includes an acceleration event that scores a car based on the time required to travel 150 ft from a resting start. The car achieving the shortest time in this event receives the most points. All cars in the competition use the same, unmodified 10 hp engine. Therefore, one of the key engineering design decisions is selecting a gear ratio that will enable a high score in the acceleration event. . The objective of this paper is to model acceleration event time and to compare predicted times with measured times.

Introduction

This introduction contains several elements of technical writing. 1. Introductory paragraph 2. literature review 3. objective statement, and 4. background theory.

The introductions starts with a broad reference to all vehicles and narrows to the topic of acceleration of an SAE Baja Car

Basic principles governing the motion of an automobile are presented by Juvinall (1983). Example problems are presented showing free body diagrams of a transmission and the forces in a drivetrain that result in forces on the wheels. A discussion of transmission choices for a Baja car is given by Kirtland (2014) which include sets of planetary gears with a shift mechanism, or a combination of a continuously varying transmission (CVT) combined with fixed planetaryor spur gears. In order to simplify driving and be successful in the other SAE Baja events (Maneuverability, Hill Climb, and Endurance Race), SAE teams typically select a CVT instead of a clutch and shift mechanism. Kirtland (2014) concludes that the preferred transmission for a Baja car is a continuously varying transmission (CVT) followed by a fixed set of spur gears.

Literature Review
Although not comprehensive, the next three paragraphs cite some literature relevant to the topic which explains information already available in the literature.

Numerous CVTs are available commercially with gear ratios that are typically in the range of 3 or 4:1. The BYU SAE Baja team in Capstone (2013) reviewed several commercially available CVTs and concluded the Gaged GX9 CVT was the best available based on gear ratio, tunability, availability, rotating mass, and overall mass. Assuming a Gaged GX9 CVT with a variable gear ratio of 3.5:0.9 is selected for a Baja vehicle, the fixed gear ratio must be selected.

Based on available literature, the overall gear ratio of Baja cars are typically near 35:1. For example, the 2012 Michigan team produced a fixed gear ratio of 10:5:1 (Michigan, 2012) and overall ratio of 36.7:1. The University of Northern Arizona (2013) utilized a 3.1:1 CVT and 12:1 fixed gear ratio or total ratio of 37.2:1.

The objective of this work is to determine the optimal fixed gear ratio to be built in combination with the Gaged GX9 CVT. This will be done by developing a model that predicts acceleration event time as a function of fixed gear ratio and testing the results of the model with timed runs of an SAE Baja car during an acceleration event.

Objective Statement
The objectives statement clarifies what will be added by the work presented in this document.

Background – Theory – Technical Information

Theory, Background, or Technical Information

Newton's second law states that the acceleration of an object is proportional to the applied forces and is inversely proportional to the mass of the object being accelerated as expressed by Equation 1.

This sections presents information that will be useful for the reader to understand the results and discussion. Some readers may skip this information but adding it

$$\vec{a} = \frac{\sum \vec{F}}{m} \tag{1}$$

A free body diagram showing the horizontal forces on an accelerating Baja car is shown in Figure 1. The torque produced by the engine is transferred through a transmission to the rear wheels producing an equal and opposite reaction force (**F**$_{\mathbf{w}}$ at the rear wheels moving the car in the forward direction. Friction and rolling resistance forces have been combined into a single force (**F**$_{\mathbf{f+r}}$) and act in the opposite direction of the wheel rotation. The final force shown is aerodynamic drag (**F**) which also acts opposite to the direction of motion.

ensures that the reader and writer have established a common knowledge base from which they can discuss and interpret the results. Various headings can be used such as Theory, Background, or Technical Information. This type of information can be a part of the **I**ntroduction as a separate section (as is the case here) or it can be located in the **M**ethods , **R**esults **a**nd/or the **D**iscussion sections. It should be placed where it makes the most sense for the reader.

Note that the figure is referenced using a number. The text refers to the figure and uses it as a part of the explanation. The reference to the figure is not an afterthought but is integrated into the flow of the text.

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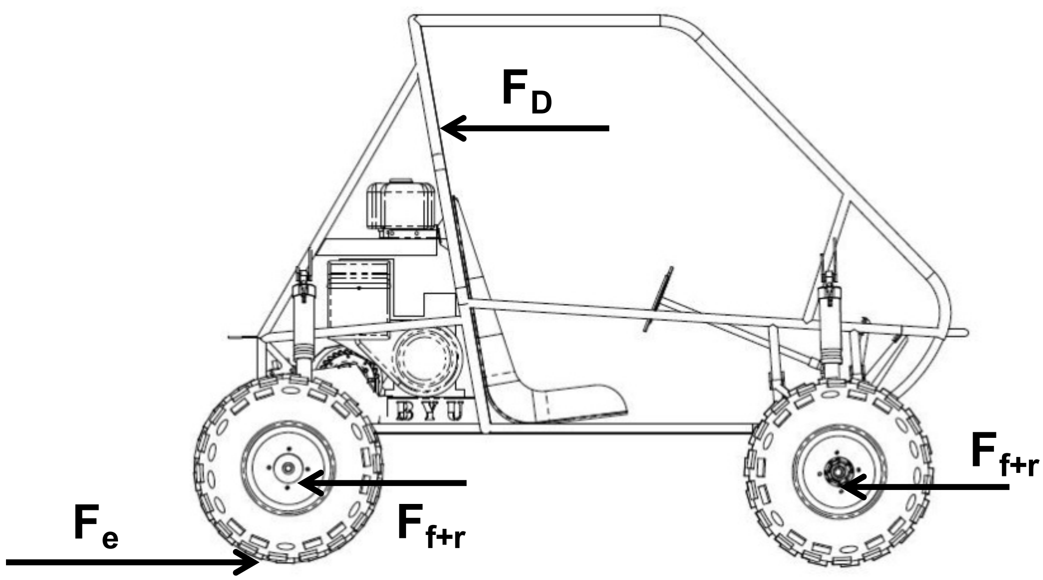

Figure 1. Free body diagram of the horizontal forces acting on an accelerating Baja car.

Figures always have a caption and that the caption is below the figure.

Equations are referenced by number.

Using the free body diagram and substituting for the various forces gives the result shown in Equation 2.

$$a = \frac{F_W - F_{f+r} - F_D}{m} \tag{2}$$

This result shows that in order to maximize acceleration; the mass should be as small as possible; the wheel force should be as large as possible; the friction, rolling resistance, and drag should be minimized.

Wheel force comes from the engine through the transmission. Engine power is the product of engine output torque and engine speed. Both quantities are limited in the Baja competition by requiring that all engines are the same size (this fixes the maximum engine torque) and all engine are governed to the same maximum speed (3600 rpm). The force at the wheels of a car are related to the torque of the engine by the Equation 3, where *T* is the engine torque, *GR* is the overall gear ratio, R_{wheel} is the radius of the rear tires and ε_d is the mechanical efficiency of the drive train.

$$F_W = \frac{T\ GR\ \epsilon_d}{R_{wheel}} \tag{3}$$

Wheel rotational speed and engine rotational speed are also related by the gear ratio as shown in Equation 4.

$$N_{engine} = N_{wheel}\ GR \tag{4}$$

Using the two equations together it can be understood that while a large gear ratio is desirable to obtain a large force at the tire, it also causes the engine speed to be high for a given wheel speed. Thus a car starting in 1st gear (high gear ratio) provides a high force at the wheel but rapidly reaches maximum engine speed after which the gear ratio needs to change before the wheel speed can increase.

A CVT shifts gears continuously with the goal of maintaining the engine at just below the governed engine speed. At the governed engine speed, the governor will reduce the output of the engine to maintain constant speed and the vehicle will no longer accelerate but will maintain a constant speed.

When two gears sets are used, a CVT and a fixed gear ratio, Equation 3 can be rewritten as shown in Equation 5 and the relationship between the engine speed and the wheel speed is changed as given be Equation 6.

$$F_{wheel} = \frac{T\ GR_{CVT} GR_{fix} \epsilon_d}{R_{wheel}} \tag{5}$$

$$N_{engine} = N_{wheel}\ GR_{CVT} GR_{fix} \tag{6}$$

When the engine is idling, wheel speed is zero and the engine speed is finite (typically 1750 rpm) which means the CVT belt which connects the engine speed to the wheel speed must be slipping or not engaged. During this period, the relationship between wheel speed and engine speed given by Equation 6 is not valid. At the start of acceleration, the engine speed increases from 1750 (idle) to 3600 rpm where the CVT begins to engage and clamp onto the belt. This causes the engine speed to slow and the wheel speed to increase until the velocity of each pulley is equal at the diameter where the belt engages. This process takes in a fraction of a second. At this point the belt is no longer slipping. The engine speed can then increases to 3800 rpm at which point the CVT must shift gear ratio to match the engine and wheel speed. Once the gear is shifted to the highest ratio, and the engine reaches the governed speed of 3800 rpm, the engine governor will cut the engine torque and not allow the engine speed to go any higher.

Methods - Experimental Method - Approach

There are two components to this work, an experimental component and a numerical model. The method used to solve the numerical model will first be explained followed by the experimental method.

Methods
This methods section contains both an experimental approach and a Modeling approach. Enough detail should be provided that others could replicate the results. Where needed, additional background information is added.

The model for obtaining the velocity and distance traveled is

Numerical Model

In order to determine the velocity (V) and distance traveled (x) in an amount of time (t) Equation 2 must be integrated twice, first to obtain the velocity and then to obtain the distance traveled. The first integral used to obtain velocity is shown in Equation 7. Since both the force on the wheel, $F_{W,}$ coming from the engine through the drivetrain and the drag force (F_D) are functions of the unknown velocity, Equation 7 can be integrated by assuming constant values for a short time period. After each time step, a new force is calculated before integrating again over a short time step, or by numerical integration. The numerical integration of Equation 7 is shown in Equation 8. The solution requires an knowledge of the three forces F_W, $F_{f,r}$, and F_D for each time step.

described in this section. The assumptions and equations associated with the model are presented.

$$N_{engine} = \frac{V}{\pi D_{wheel}} \, GR_{CVT} GR_{fix} \tag{7}$$

$$\int_{V_0}^{V} dV = \int_0^t \left[\frac{F_W - F_{f+r} - F_D}{m}\right] dt \tag{8}$$

$$V_{i+1} = V_i + \left[\frac{F_W - F_{f+r} - F_D}{m}\right] \Delta t_i \tag{9}$$

The initial and subsequent values for each force at each time step are found as follows:

$\underline{F_W}$ – The wheel force was found by using Equation 5 which requires the Torque. A torque curve for a Briggs and Stratton Intek 305 engine measured by Kirtland (2014) is shown in Figure 2 and curve fit Equation 10 as determined by Tanner (2016). The torque curve is seen to be relatively flat or insensitive to the engine speed. For simplicity, the torque from the engine was assumed to equal the torque at 3600 rpm (12.96 ft-lbf) as long as the engine speed did not exceed 3800 rpm. Once the engine speed exceeded 3800 rpm the torque was made zero until the engine speed dropped below 3800 rpm. During initial acceleration, some of the

Note the citations. Citations can occur anywhere in the document. They are necessary in all documents regardless of how formal or abbreviated when the work of others is utilized.

engine torque will be lost as the belt slips and therefore the model will tend to over predict the torque applied to the drivetrain and under predict acceleration times. A measurement of engine speed vs. time will be used to evaluate the accuracy of the constant torque assumption.

The mechanical efficiency, ε_d, of the drivetrain was taken to 0.72 based on the estimate of Tanner (2016). The other parameters in the equation are geometric parameters if the car.

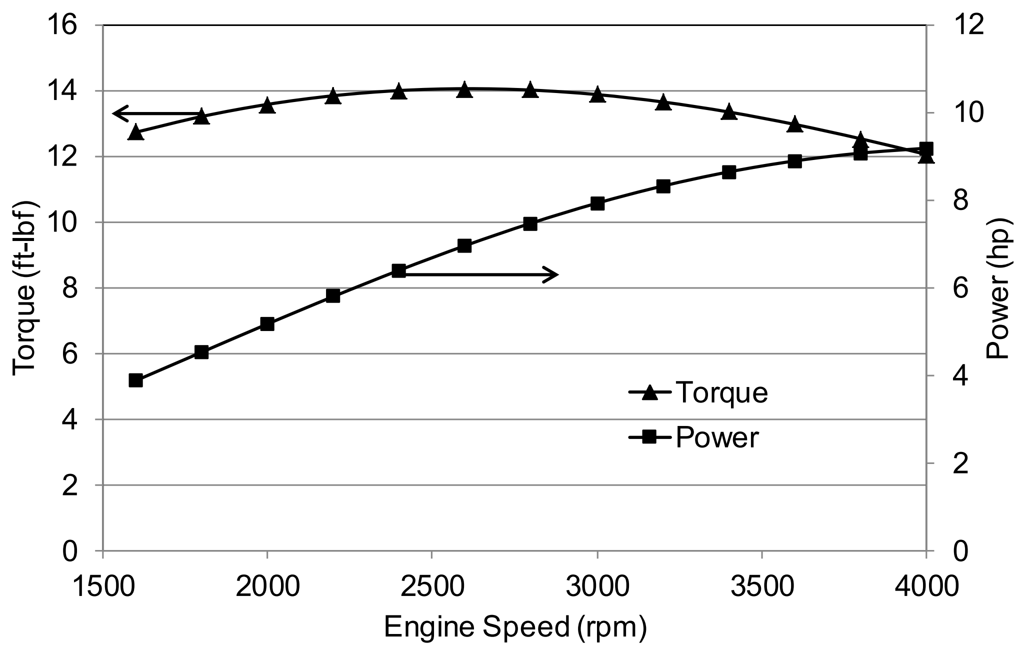

Figure 2. Torque curve of Briggs and Stratton Intek 305 Engine from Kirtland (2014)

$$T = \frac{(N_e - 800)^2}{700{,}000} + 0.002 N_e exp\left(\frac{N_e - 3700}{800}\right) + 9.51 \tag{10}$$

F_{f+r} – The friction plus rolling resistance for the car was estimated by determining the force required to keep a Baja car moving at a slow constant speed and was found to be 18 ft-lbf.

F_D – Was modeled as shown in Equation 11 where ρ is the density of air (found using the ideal gas law), V is the velocity of the vehicle, A is the frontal area of the vehicle (estimated to be 1.09 m^2), and C_D is the drag coefficient (1.9 used for a rectangle in cross flow).

$$F_D = \frac{1}{2}\rho A V^2 C_D \tag{11}$$

The distance traveled was determined by numerical integration as shown by Equation 11. The velocity is zero at the beginning of the acceleration event and updated before taking the next time step.

$$X_{i+1} = X_i + V_i \, \Delta t \tag{12}$$

Before showing modeling results, the experimental method section is next. It is an option to present the results from modeling here before going into experimental methods but in this case all of the methods are being presented before any of the results.

Experimental Approach

Two experiments were performed; one to determine the time to complete an acceleration event and one to measured engine speed during an acceleration event. The later measurement will be used to determine if the engine torque is modeled correctly during CVT engagement.

Experimental Approach

This section explains how the experiments were performed.

Acceleration Test

Vehicle and driver mass were measured prior to the acceleration runs and found to be 450 lbm. A flat piece of pavement was identified with a start and finish line market at a distance of 150 ft. Electronic start and finish instrumentation was not available and so timing of the vehicle was done using a stop watch and visual cues. When the timers were ready, the driver pressed the accelerator pedal completely to the pedal stop to begin accelerating. An assistant standing next to the vehicle dropped his hand at the instant he perceived the vehicle moved in the forward direction. Timers (3 people) standing at the finish line, 150 ft from the start line, began timing as soon as they saw the hand drop from the driver assistant. They stopped timing when the car reached the finish line.

Engine Speed vs. Time A second experiment included the placement of an inductive coil around the spark plug connected to a capacitor and data acquisition (DAQ) system as shown in Figure 3. Data were recorded from the circuit at a rate of 200 Hz. The number of pulses measured in a given amount of time was used to determine the engine speed. The uncertainty in the number of pulses collected was ± 1 pulse. The number of pulses occurring over a period of time is n = N(rpm)*t/60, where t is the measurement time used to evaluate the engine speed. At maximum engine speed (3600 rpm) and a sample time of 0.5 sec, the uncertainty of the measurement is 1/30 or 3.3% or 120 rpm.

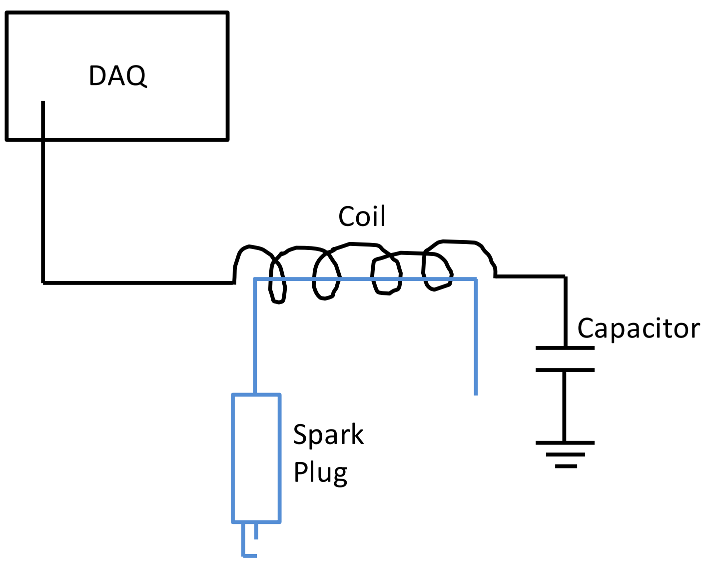

Figure 3. Wiring diagram of data induction coil used to indicate spark timing

Results – Discussion - Discussion of Results

Measurement Results

Results for the acceleration test are shown in Table 1. The average acceleration time was measured to be 6.03 s with a standard deviation of 0.135 s. In spite of

Results and Discussion

This section contains results and a discussion of the results. The discussion of results might also contain literature citations, and theory or background information, In some cases the results may lead to additional modeling or measurements requiring methods to be explained.

the relatively crude method for measuring time, the standard deviation suggests a 95% uncertainty of ± 0.270 s. The deviation in measured times results from human measurement error and driver repeatability.

Table 1. Raw data of recorded time (seconds) for car to travel 150 ft.

Timer	Run 1	Run 2	Run 3	Run 4
Timer 1	6.1	6.0	6.0	6.2
Timer 2	5.9	6.1	6.0	6.0
Timer 3	6.0	5.7	6.2	6.1
Average	6.03			
Std. Dev.	0.135			

Modeling Results

A plot of the predicted time required to complete the acceleration event as a a function of the fixed gear ratio is shown in Figure 4. The time to reach 150 ft. initially decreases with increasing gear ratio and then increases. A minimum occurs at a fixed gear ratio of 8:1. An optimum occurs because at high gear ratio the car accelerates rapidly but quickly reaches a relatively low top speed limited by the engine. At a low gear ratio the car accelerates slowly and is still accelerating after traveling 150 ft.

Tables and Figures allow a reader to find and evaluate results easily.

A table contains a heading and a number. The heading of a table is always at the top or above the table. This is the opposite of a figure which is labeled below.

This tables allows the reader to see how the data were taken (3 timers, 4 runs) as well as the variation in the data between timers and between runs. The reader can draw their own conclusions on the data if needed.

The table is referenced by number. The table is referenced at the beginning of the discussion allowing the reader to look and the data before it is discussed. Results in the table are referenced in the discussion. The reference to the table is not an afterthought done at the end after the discussion is complete.

Discussion

Discussion can occur during the presentation of the data and after the presentation.

The initial discussion reviews important information contained in the table and graph drawing attention to details the writer feels are important.

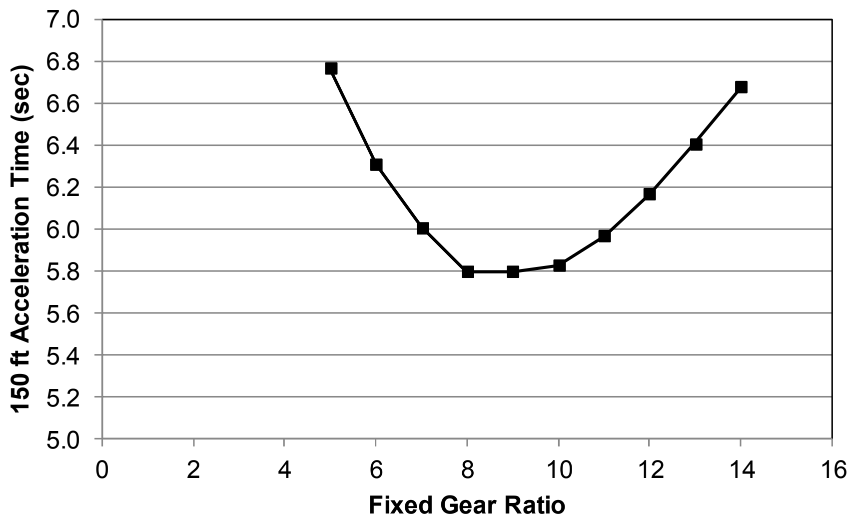

Figure 4. Time required for a car initially at rest to travel 150 ft as a function of the Fixed Gear Ratio connected to a CVT with ratio 3.5:0.9.

Discussion

The measured and predicted results agree well. The modeled predicted a time of 5.8 s while the measured value was 6.0 ± 0.27 s. . This difference was within the 95% confidence interval of the measurement variability. The model is therefore demonstrated to be accurate at this single operating condition measured.

In order to evaluate the model for additional operating conditions, the validity of the assumptions made will be discussed. One assumption was that the engine torque was constant over the acceleration period as long as the engine speed did not exceed the governed speed of 3800 rpm. The engine speed vs. time is shown in Figure 5.

Two sets of data are represented in the figure. A signal was collected from the coiled wire at a sampling frequency of 200 Hz. A pulse would occur in the recorded signal each time the spark plug fired. The engine speed was determined at time "t" by counting the number of times the voltage passed above an arbitrary threshold over a known time period of t + Δt. The rpm data are seen to change in discrete intervals as the number of pulses counted changed by plus or minus one pulse. The raw data were smoothed (by simply averaging 10 data points) to

Discussion
After describing the data, the author interprets the data providing context and meaning. It is often critical to review assumptions and uncertainty in this section so that the data can be viewed in the proper context. Previous results in the literature and consistency with the theory and background should be discussed. Based on this analysis of the data, the author draws meaningful conclusions.

produce the final engine speed curve shown by the solid black line.

The engine speed begins near 1750 rpm which is the idle speed. At approximately 0.5 sec. the acceleration event begins and the engine speed increases rapidly to 3500 rpm at 1.4 sec. During this period the engine, CVT, and wheel speed are accelerating while the gear ratio remains fixed. At 3500 rpm the engine speed flattens and drops slightly over the next 5 seconds from 3550 to 3000 rpm. During this period the CVT is shifting and causing the gear ratio to decrease. The wheel speed (not shown) is increasing while the engine speed decreases slightly. Shortly after 7 seconds, the engine speed drops rapidly indicating the acceleration run is complete and the engine slows back down to the idle speed.

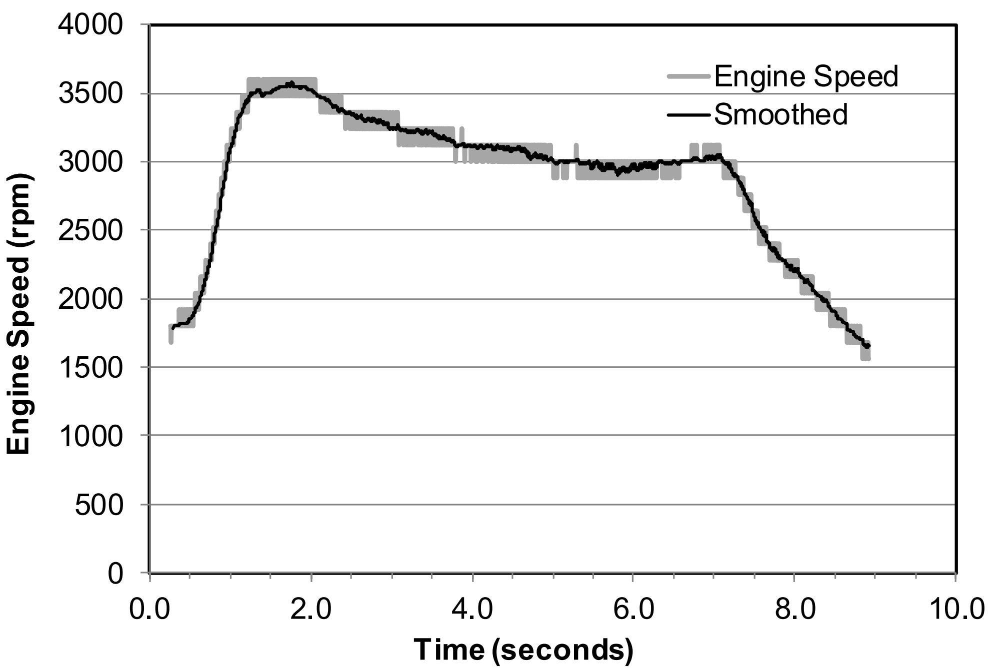

Figure 5. Engine speed as a function of time during an acceleration run.

The engine speed data shows that the assumption of a constant torque used in the model is relatively good except for the period at start-up when the engine is accelerating and the belt is slipping. This period appears to last approximately 0.9 seconds. During this period the torque applied to the wheels is actually only a fraction of the engine torque. This means the model should under predict the acceleration time because torque is over predicted. At most the time could be under predicted by 0.9 s. During the rest of the acceleration event, the engine speed indicates the torque should be between 13 and 14 ft-lbf. This is slightly higher than the torque that was used 12.96 ft-lbf which would cause the predicted time to be longer than the measured time.

Several other assumptions in the model should also be considered.
The model did not include rotational inertia (the energy needed to accelerate the rotating members of the car) including the engine, CVT and wheels. Adding inertia would increase the time predicted by the model. Finally, the model assumed a constant friction and rolling resistance. I reality a large component of the friction is dependent on speed and increases with speed. For example, bearing friction will increase with increasing rotational speed.

Conclusions

The conclusions section is often grouped under the Results and Discussion part of IMRAD but is set apart in a separate section in this and most documents. Isolating a section allows a focus on the conclusion and recommendations. All conclusions and recommendations should be supported by results and discussion. This section may repeat what is stated in the discussion. It should not contain new information. It is a common mistake to make a statement here not supported by facts in the document.

Summary and Conclusions

A numerical model of an SAE Baja acceleration event has been completed in order to predict the time of acceleration and the optimal fixed gear ratio to be used with a CVT of 3.5:0.9 ratio. Acceleration times were measured and compared to the model for a single fixed gear ratio. Engine speed as a function of time during the acceleration event was also measured to determine the validity of a constant engine speed assumption in the model.

The models suggested that a fixed gear ratio of 8:1 would be optimal for producing the lowest acceleration times when combined with a CVT of 3.5:0.9 variable gear ratio. The model also suggests that gear ratios from 8:1 to 10:1 will produce similar acceleration times. The modeled acceleration time 5.8 s was within the 0.270 s uncertainty of the measured acceleration times indicating the model was accurate for the case measured. Assumptions used in the model

References

This section may be called a bibliography, references or sources cited. The format or ordering of information for each citation should be as suggest the model may be as much as 0.9 s in error or more because the model does not account for the transient startup period when the CVT belt is slipping, rotational inertia, and friction as a function of component rotational speed. This suggests other values in the model such as rolling resistance and mechanical efficiency may be incorrect but are compensating for the other errors in the model. Based on the model results it is recommended that a fixed gear ratio between 8:1 and 10:1 be used when the CVT of 3.5:0.9 is selected.

requested by person for whom the document is written. The same format should be used for all of the references. If no format is specified pick a format from the ASME journal or another trusted source and remain consistent.

It has become common to add the URL of references to the end of the citation. In some cases this is requested or required.

References

1. Juvinall, R. C., *Fundamental of Machine Component Design*, Wiley, 1983.

2. Kirtland, Dakoda, "2014 University of Cincinnati Baja SAE Drivetrain,", Bachelorette Thesis, University of Cincinnati, 2014. https://drc.libraries.uc.edu/handle/2374.UC/731831

3. Ellis, D., Delimont, I., Taysom, N., and Bleazard, T., BYU Car 47 SAE Baja Design Report, Brigham Young University, 2013.

4. Ebsch, E, Kudla, J., O'Brian C., Quick, B., Baja Gear Reduction, ME 450 Final Report, (2012). https://deepblue.lib.umich.edu/handle/2027.42/96183

5. Almuflih, A., Perryman, A., Ming, C., Zhu, Z., Pan R,. SAE Mini Baja Drivetrain, Department of Mechanical Engineering, Northern Arizona University, 2013.
http://www.cefns.nau.edu/capstone/d4p/index.php