An Example of the BYU MeEn Real World Problem Solving Process

Carl Sorensen | Version 1.1 21 Aug 2017

This document illustrates how the steps of the BYU MeEn Real World Problem Solving Process could be applied. The process consists of 4 steps:

1. Develop a set of engineering problem statements related to the real-world problem
2. Solve the engineering problems to obtain an engineering model
3. Explore the effects of various assumptions and values on your model solutions
4. Communicate ideal answers for engineering problems

Each of these steps is illustrated below. The real world problem is to select appropriate tie-down straps to hold a canoe to a roof rack on an automobile.

DEVELOP

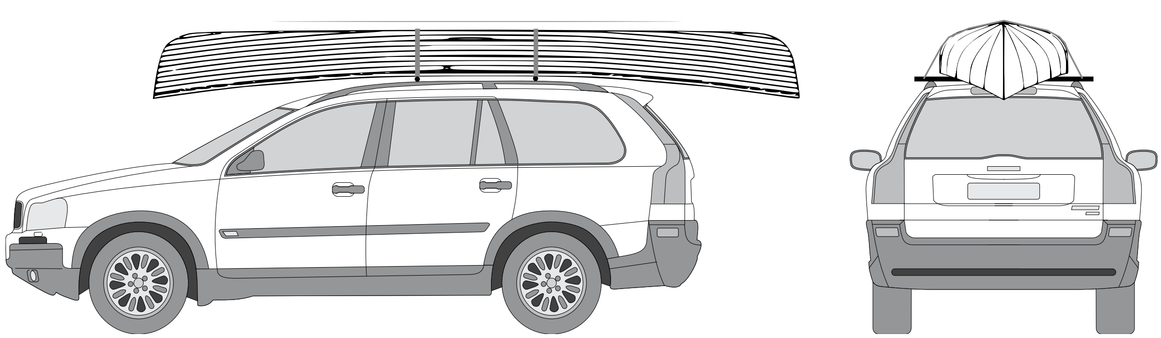
Use the 5 Ps to develop a problem statement

Principles

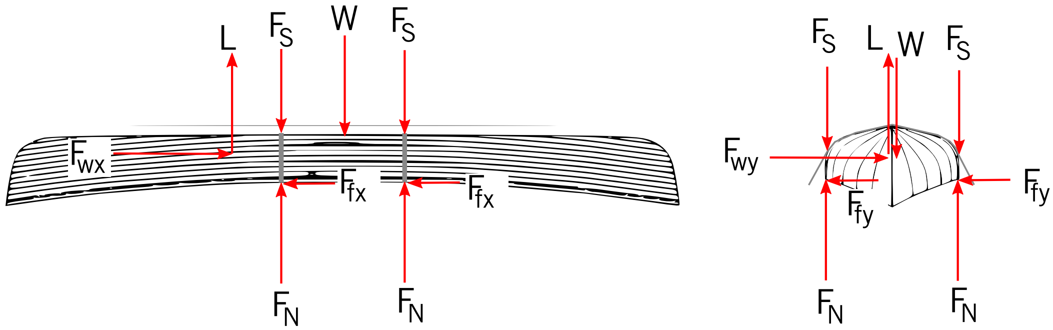
Static friction: $F = \mu N$
Aerodynamic drag: $F = ½\rho v^2 A C_D$

Static equilibrium: $\sum F = 0$
Inertial loads during braking, accelerating, cornering.
Impact loads with trees or brush

Pictures



Sketch of canoe tied to roof and mounted to rack with hold down straps.



Free body diagrams of canoe.

Possiblities

CFD to determine aerodynamic forces
Simple approximations to estimate aerodynamic forces
Full-scale tests to measure aerodynamic forces
Scale-model tests to measure aerodynamic forces
Handbooks for μ values
Tests for μ values
Stated strengths for ropes/straps
Measured strengths for ropes/straps
Calculated strengths for ropes/straps

Precision

We can almost do order-of-magnitude solution, because there is no penalty for going too big.
Certainly one significant figure is sufficient
This leads to the idea that a relatively simple model can be used.

Performance

Aerodynamic drag force on the canoe
Friction force on the canoe
Is aerodynamic lift force important?

Problem Statement

To solve this situation, we will probably use a series of coupled problems. Each is listed below.

A canoe is to be strapped on a roof rack on an automobile. There are two tie down straps going from one side of the canoe to the other. Each tie down strap is connected to a canoe support that is attached to the roof rack. The straps keep the canoe in place by means of friction between the gunwales of the canoe and the roof rack.

A) If the tension in the strap is T, the weight of the canoe is W, the aerodynamic lift on the canoe is L, and the static coefficient of friction between the gunwales and the rack is μ, determine the maximum total frictional force that the rack can apply to the canoe.

B) For a wind of speed $v_x$in the longitudinal (x) direction, if the cross sectional area of the canoe is A_x and the coefficient of drag is C_{Dx}, what is the longitudinal force applied to the canoe (F_{wx})?

C) For a wind of speed $v_y$in the transverse (y) direction, if the cross sectional area of the canoe is A_y and the coefficient of drag is C_{Dy}, what is the lateral force applied to the canoe (F_{wy})?

D) For a vehicle traveling at a maximum speed of 80 mph, with a maximum wind speed of 40 mph, what strap tension is necessary to ensure that the canoe stays rigidly attached to the rack (assume that the lift force is neglible)?

SOLVE

Solve the problem set above

Summary

The overall problem set is aimed at determining the strap tension necessary to hold the canoe in place. The subproblems include identifying the relationship between strap tension and the maximum frictional force, the relationship between lateral and longitudinal wind speed and lateral an longitudinal drag force, and the relationships between longitudinal and lateral forces and total frictional force.

Each of these problems is relatively straightforward.

Approach and Assumptions

We will approach the aerodynamic problems using the definition of drag coefficient.
We approach the frictional problem using the definition of static friction.
As we do not know exact values of μ and C_D, we will select approximate values that are reasonable. We will measure the areas from a drawing of a typical canoe. We will estimate weight from available data on typical canoes.
From a measurement: A_x is 0.25 m^2 and A_y is 1.4 m^2
W is 35 lb.

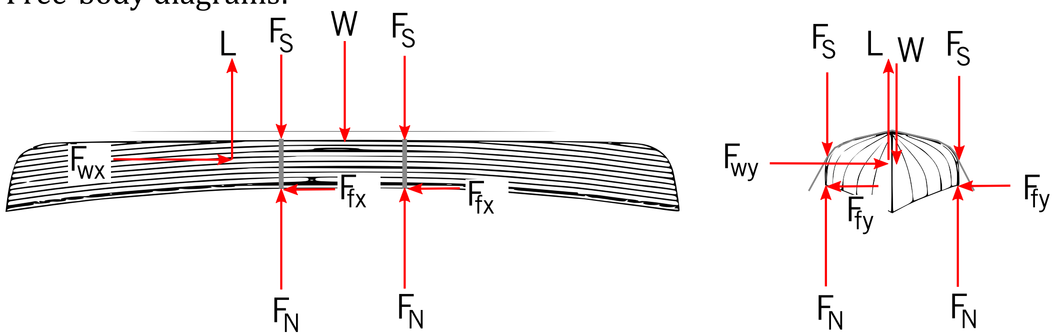
Assumptions:
The density of air is 1.2 kg/m^3
The lift force is negligible
The weight of the straps is negligible.
The center of pressure is at the center of mass (only necessary if lift is not negligible).
Both straps have equal tension T.
The contact between the straps and the canoe is approximately frictionless.

The angle that the straps make with the rack is negligible (i.e. all the tension is straight down).

Framework

Given: Maximum wind speed v_w of 40 mph. Maximum vehicle speed v of 80 mph.

Free-body diagrams:



Execute

First, we consider the effects of the straps on the frictional force preventing the canoe from moving.

$$F_s = 2T$$

$$\sum F_z = 0;\ -2F_s - W + 2F_n + L = 0$$

$$F_n = (2F_s + W - L)/2$$

$$F_{fmax} = 2\mu F_n$$

For L=0,

$$T = \left(\frac{F_{fmax}}{\mu} - W\right)/4$$

This equation only holds if F_{fmax}/μ is greater than W. Otherwise, the weight of the canoe alone is sufficient to hold it in place.

We now consider the force due to the relative wind seen by the canoe. There is no simple calculation for the wind force in an arbitrary direction. Such a calculation could be performed using Computational Fluid Dynamics (CFD), but CFD is beyond the scope of this problem. Therefore, we will assume the worst-case longitudinal force and the worst-case transverse force, and add these forces together vectorially. We recognize that this combination of worst-case forces cannot happen simultaneously; nevertheless, this is a simple calculation that is conservative in the sense that the calculated force should exceed the actual force. Thus, a tension based on this force will be greater than the actual tension needed.

For the longitudinal force:

$$F_{wx} = ½\rho v_{wx}^2 A_x C_{Dx}$$

The maximum force will occur when the wind is a headwind, so

$$v_{wx} = v + v_w$$

$$F_{wx} = ½\rho(v + v_w)^2 A_x C_{Dx}$$

For the transverse force:

$$F_{wy} = ½\rho v_{wy}^2 A_y C_{Dy}$$

The maximum force will occur when the wind is directly from the side, so

$$v_{wy} = v_w$$

$$F_{wy} = ½\rho(v_w)^2 A_y C_{Dy}$$

Combining these two worst-case forces as vectors gives the worst-case wind force:

$$F_w = \sqrt{F_{wx}^2 + F_{wy}^2}$$

To hold the canoe securely, F_w must be less than or equal to F_{fmax}

$$T = \frac{\left(\frac{F_{fmax}}{\mu} - W\right)}{4}$$

Reflect and Report

The units are consistent – both sides have units of force.
The equation makes physical sense – both the tension and the weight of the canoe have the same sign, and the weight has ¼ the contribution of the tension. This makes sense because there are four strap connection points, each with T, and only one weight.

EXPLORE

Use the ConVerSAnT method

Construct

The equations were implemented in a MATLAB function. Here is the code:

```
function [ MinT,fw ] = StrapTension( v, vw, ax, ay, cdx, cdy, w, mu )
%StrapTension: Calculate the minimum tension necessary in a canoe
%              hold down strap to prevent a canoe from moving on a
%              roof rack given a specified aerodynamic load.
%
%   Note: this is a worst-case calculation; more accurate calculations
%   would require CFD simulations

% Input arguments:
%   v: car velocity in miles/hour
```

```
%   vw : wind speed (relative to ground) in miles/hour
%   theta: angle between car velocity and wind direction in degrees
%   ax, ay : cross-sectional area of canoe in travel direction and
%             lateral direction, respectively.  Units are m^2
%   cdx, cdy: coefficient of drag in travel direction and lateral direction
%   w: weight of canoe in lbs
%   mu : coefficient of friction between canoe and rack

% Outputs:
%   MinT: Minimum tension in strap (N)
%   fw: magnitude of wind force (N)
%   phi: angle of wind force with respect to vehicle direction

% Copyright 2016 Carl D. Sorensen
% This code may be freely copied and used for any desired purpose


rho = 1.2; % density of air in kg/m^3

mph2mps = @(v) v*1609.344/3600; % convert from mph to m/s
lb2n = @(w) w * 4.44822; % convert from lbf to N

% get all units to SI
v = mph2mps(v);
vw = mph2mps(vw);
w = lb2n(w);

vwx = v+vw;
vwy = vw;

fwx = rho*vwx.^2.*ax.*cdx/2;
fwy = rho*vwy.^2.*ay.*cdy/2;

fw = sqrt(fwx.^2+fwy.^2);

MinT = (fw./mu-w)/4;
MinT(MinT<0)=0; % Set minimum values to zero

end
```

Verify and Validate

For v = vw = 0, T should be 0.

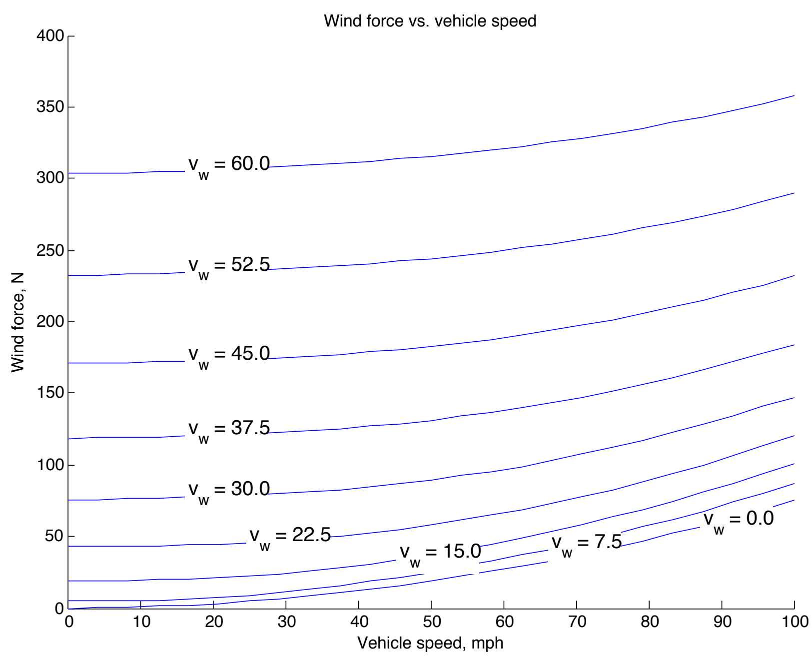
```
>> W

W =

    35

>> mu

mu =

    0.4000

>> Ax

Ax =

    0.2500

>> Ay

Ay =

    1.4000
```

```
>> Cdx

Cdx =

    0.2500

>> Cdy

Cdy =

    0.5000

>> [T,f]=StrapTension(0,0,Ax,Ay,Cdx,Cdy,W,mu)

T =

     0
```

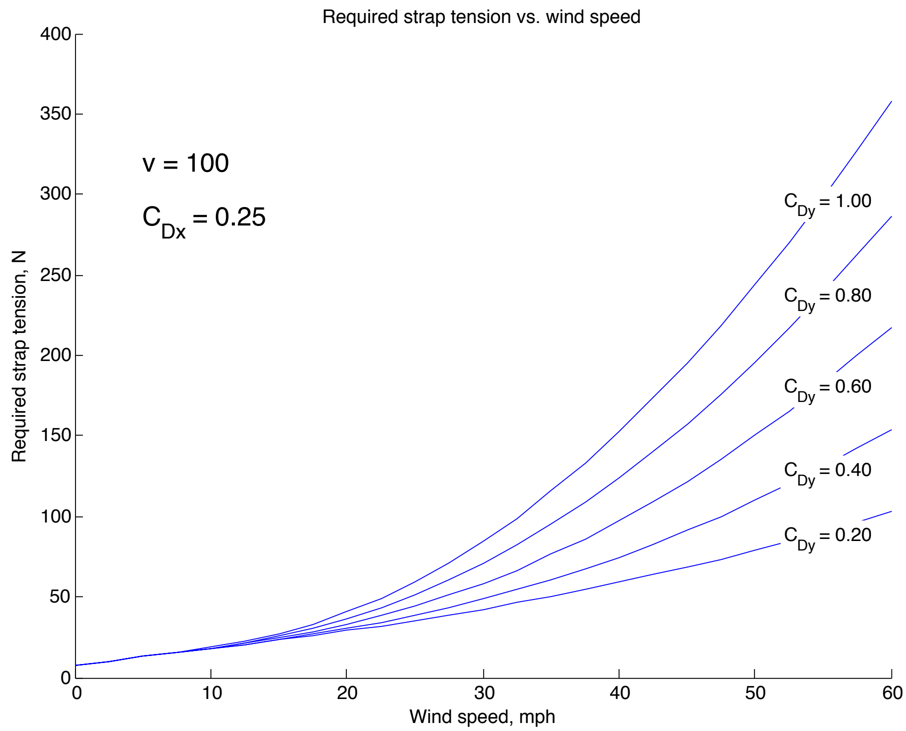
Search

I'll make a series of plots including the following:

1) Worst case force versus vehicle speed at various wind speeds.



2) Tension versus wind speed for the maximum vehicle speed and various CDY values



Matlab code for generating the plots is shown here:

```
% Make plots for canoe force and strap tensioy

% Canoe parameters
Ax=0.25; % frontal area, m^2
Ay = 1.4; % lateral area, m^2
Cdx = 0.25; % longitudinal drag coefficient
Cdy = 0.5; % lateral drag coefficient
W = 35; %Weight, lbs
mu = 0.4; %Coefficient of friction

vwmax = 40; % maximum wind speed, mph
vmax = 100; % maximum vehicle speed, mph
vel_count = 25;
vel  = linspace(0,vmax,vel_count);
wind_count = 9;
vwind = linspace(0,1.5*vwmax,wind_count);

% Plot of various forces
figure(2);
clf;
hold on;
title('Wind force vs. vehicle speed');
xlabel ('Vehicle speed, mph');
ylabel('Wind force, N');
start_label_point = 22;
delta_label_point = 5;
 for curve = 1:wind_count
    [T,f]=StrapTension(vel,vwind(curve),Ax,Ay,Cdx,Cdy,W,mu);
    plot (vel,f);
    label_point = start_label_point-(curve-1)*delta_label_point;
    if (label_point < 5)
        label_point = 5;
    end
    text(vel(label_point),f(label_point), ...
        sprintf('v_w = %3.1f',vwind(curve)), ...
        'FontSize',12,'BackgroundColor','white');
```

```
end;


% Plot of various tensions
figure(3);
clf;
hold on;
title('Required strap tension vs. wind speed');
xlabel ('Wind speed, mph');
ylabel('Required strap tension, N');
label_point = 23;
wind_count = 25;
vwind = linspace(0,1.5*vwmax,wind_count);
C_dy = linspace(0.2,1,curve_count);
Tmax(wind_count) = 0;
for curve = 1:curve_count
    for wind = 1:wind_count
        [T,f]=StrapTension(vmax,vwind(wind),Ax,Ay,Cdx,C_dy(curve),W,mu);
        Tmax(wind) = max(T);
        Tm = max(T)
    end;
    plot(vwind,Tmax);
    text(vwind(label_point),Tmax(label_point), ...
        sprintf('C_{Dy} = %0.2f',C_dy(curve)), ...
        'FontSize',10,'BackgroundColor','white', ...
        'HorizontalAlignment','center', 'VerticalAlignment','cap');

end;
text(5,300,{sprintf('v = %d',vmax),'',sprintf('C_{Dx} = %0.2f',Cdx)},...
    'FontSize',14);
```

Analyze

Because of the larger transverse area and the larger transverse C_D, the wind speed dominates the force requirement. At a wind speed of 45 miles/hour and a vehicle speed of 80 miles per hour, the vehicle speed is responsible for about 15% of the total force. Thus, it's the wind that we need to worry about the most.

The aerodynamic forces are relatively small in all of these situations, however. Even at an unrealistic transverse C_D of 1.0, and a windspeed of 60 mph, the required tension is only 350 N. A 1 inch polyester web strap has a working load of 3700 N, or more than 10 times the required tension. A 1 inch polypropylene strap has a working load of 1175 N, or more than 3 times the required tension.

This calculation has ignored lift force. If there is a significant lift force, the tension required will increase. Perhaps a CFD, wind tunnel test, or real-world experiment should be performed to measure the lift force.

Translate

A 1-inch wide tie down strap of either polyester or polypropylene will carry the required tension to hold the canoe, even in extreme conditions.

COMMUNICATE

See the memo on the next page for a sample of communication. Note that this memo follows the IMRaD format as recommended by BYU Mechanical Engineering, but with the recommendation at the beginning of the memo.

Strap selection for canoe tie down

20 July 2016

Recommendation

Based on a worst-case aerodynamic drag analysis we recommend that the canoe be held to the rack with two one-inch wide polyester or polypropylene straps. The straps should have a ratchet clamp in order to develop a tension of about 50 lbs. This tension should be enough to prevent motion of the canoe relative to the rack.

Introduction

A simple aerodynamic drag model was used to determine the size of strap necessary to hold a canoe on a roof rack. The canoe is to be strapped to the rack with two straps. Each strap is connected at both sides of the rack and passes laterally over the canoe, using the tension in the strap to secure the canoe to the rack.

Methods

The longitudinal and cross-sectional area of the canoe were measured to be 0.25 and 1.4 m^2, respectively. Based on comparisons with standard shapes, the longitudinal and transverse drag coefficients were estimated to be 0.25 and 0.5, respectively.

For a worst-case loading, the aerodynamic force due to a headwind of 40 mph coupled with a vehicle speed of 80 mph was added to the aerodynamic force due to a crosswind of 40 mph with a vehicle speed of zero.

Aerodynamic forces were given by

$$F = ½\rho v^2 A C_D$$

with the appropriate values for v, A, and C_D as described previously.

The canoe was modeled as being held to the rack with two straps (four attachment points) that had constant tension T. The lateral force resistance at each contact point is given by the product of the coefficient of friction and the normal contact force. So the total lateral force resistance is given by

$$F_{fmax} = \mu(4T + W)$$

Results

Figure 1 shows the required strap tension for a vehicle speed of 100 mph and various wind speeds. For a wind speed of 40 mph and a tansverse C_D of 0.6, the required strap tension is about 100 N (22 lb). Since the safe working load of a 1 inch strap is 1175 N, the 1 inch strap has sufficient strength for this application.

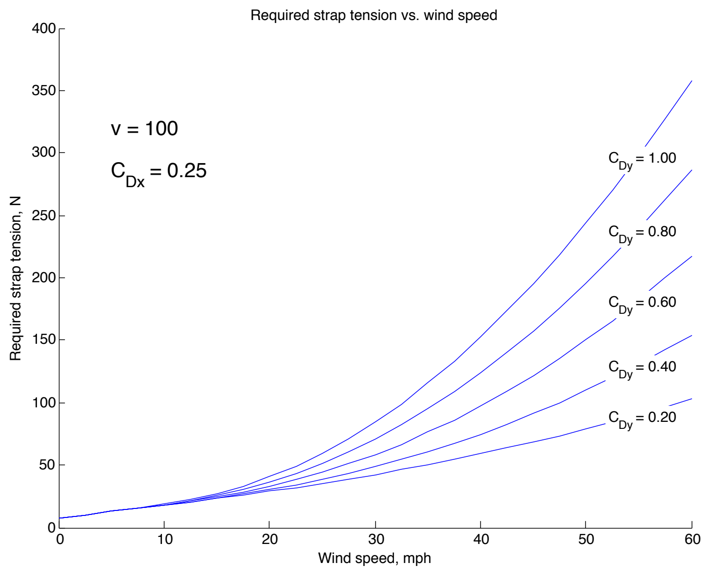


Figure 1: Required strap tension for various wind speeds and transverse drag coefficients, with a vehicle speed of 100 mph and a longitudinal drag coefficient of 0.25

Discussion

This solution technique should be conservative, but we have not measured the actual drag coefficients or the actual force on the canoe. If more information is needed, a CFD model or physical experiments could be performed. However, there is sufficient margin of strength in the straps that it is not believed these steps are necessary.

This solution neglected any aerodynamic lift or downforce on the canoe. If there is significant lift, the required tension would be increased. Previous experience with canoes on roof racks causes us to believe that the lift is negligible, particularly given the large margin of strength in the straps.